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The analyses show that the maximum length deep flaw and maximum length shallow flaw produce nearly equal CTOD values under loading. The very shallow flaw (a/t=0.125) produced very low CTOD values (<0.005 in.) at wall strains of 2*\( \varepsilon \). The Net Ligament Yield condition is reached at CTOD values <0.005 in. in each case. The finite element and experimental results confirm the fundamental approach for flaw assessment in Appendix A for both long and shallow flaws. The application of Fig. A5 with critical CTOD values obtained from deeply notched bend bars is conservative. The degree of conservatism varies with crack size.

Fracture mechanics, nonlinear, elastic-plastic, finite elements, Crack Tip Opening Displacements (CTOD) experimental tests, API-1104, BSI-PD6493, pipelines, circumferential cracks, bending loads, three-dimensional analyses

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CHAPTER 1

INTRODUCTION

1.1. GENERAL

The American, British, and Canadian workmanship standards, API-1104:1983 [1], BSI-4515:1984 [8], and CSA-Z184:1983 [9], have been traditionally applied to determine the acceptance of girth welds in pipelines. The workmanship standards do not reflect the fracture resistant properties of the pipeline material or the type and magnitude of loadings applied to the pipeline. To provide an engineering based approach, these standards are being extended to include alternate acceptance criteria for defects in pipeline girth welds that exceed the workmanship criteria. Based on the concepts of nonlinear fracture mechanics, the alternate procedures incorporate the material’s fracture toughness and in-service loadings to facilitate a more realistic fitness-for-purpose evaluation of a girth weld defect.

Part-through circumferential flaws in the girth welds, for example those caused by lack of sidewall fusion, are of most concern when the pipeline experiences axial tensile stresses. Axial stresses may arise from bending caused by misalignment and fit-up, thermal fluctuations, subsidence, washouts and laying operations off barges. These loadings can induce large amplitude stresses but they have a low frequency of occurrence. Consequently, the potential failure modes include plastic collapse, ductile fracture, and brittle fracture rather than crack extension due to fatigue.

Attention became focused on the fracture behavior of relatively long, shallow flaws by a series of tests on pipeline girth welds conducted at the Welding Institute of Canada (WIC) [19,20]. For several specimens, the measured failure strain was equal to or only slightly greater than the failure strain predicted using the design curve procedures described in BSI-PD6493 [6]. PD6493 is intended to have a safety factor of at least 2. These tests lead to questions about the adequacy of the procedures in API-1104:Appendix A for long, shallow flaws in pipeline girth welds. Appendix A is derived from PD6493 by specialization to certain defect sizes/types and material toughness.
levels typically found in pipeline construction.

The long, shallow flaw in a girth weld tests the limits of applicability of PD6493. First, PD6493 includes a partly empirical procedure based on equivalent stress-intensity factors ($K_I$) for through-cracks in tensile panels to compute an applied Crack Tip Opening Displacement (CTOD). This simplification may not be reasonable for long, circumferentially oriented flaws in pipes. Second, shallow flaws are found to require an applied axial stress that approaches the yield stress to develop CTOD values in the range of minimum acceptable toughness levels (0.005 in. to 0.010 in.). Use of the design curve in PD6493 at nominal stress levels approaching the yield stress is known [4] to be inaccurate due to net ligament yielding. However, recategorization of the long, shallow flaw as a full length through-crack, which is required in PD6493 for stress levels approaching yield, leads to unrealistically low tolerable stress levels.

To address these concerns on the magnitudes of crack driving force, 3-D nonlinear finite element analyses are conducted in this study to predict applied CTOD values for part-through circumferential cracks in pipes subjected to bending. The surface cracks are located in the base metal and on the external surface of the pipe. Figure 1 illustrates the general arrangement of the pipe section modeled in the analyses. Three semi-elliptical cracks representing short, deep flaws and long, shallow flaws are considered. Emphasis is placed on the elastic-plastic behavior of pipes containing flaws with $a/t \to 0.25$ and $L/D \to 0.4$, where $a$ is the crack depth, $t$ is the wall thickness, $L$ is the crack length and $D$ is the pipe diameter.

To provide experimental confirmation of the finite element results, one test was conducted for a pipe containing a long, shallow surface flaw located in the base metal. The Welding Institute of Canada performed the test under contract to API. The tested pipe, fabricated from X-60 material, had a 36 in. diameter and 0.9 in. wall thickness. An external surface crack of dimensions $a/t = 0.13$ and $L/D = 0.38$ was placed in the pipe wall. The pipe was loaded to failure in bending at room temperature to allow development of large plastic zones. Instrumentation placed on the tested pipe, including near crack and remote strain gages and CMOD gages, recorded data for comparison with finite element results.
The numerical results obtained in this study are discussed relative to the provisions of API-1104:Appendix A for assessment of surface flaws in girth welds. Computed values of CTOD obtained from the finite element solutions are compared to values given in API-1104:Appendix A. The comparison provides a measure of the conservatism for the crack driving force reflected in API-1104:Appendix A. The degree of conservatism on crack driving force represents one contribution to the total safety factor against failure by fracture obtained using Appendix A. Other factors including the influence of test specimen geometry to measure the material fracture toughness and weld overmatching are briefly discussed.

1.2. BACKGROUND

The fracture mechanics procedures proposed for the API, BSI, and CSA standards adopt the Crack Tip Opening Displacement (CTOD) as the parameter to characterize the material toughness and driving force for fracture. The general methodology parallels that described in British Standards Institute document PD 6493 with specialization to certain categories of welding processes, defect size and type, and levels of material toughness typically found in pipeline construction. Simplified stress analyses are employed to estimate the CTOD present in the flawed pipe for a specified level of axial stress (or strain).

The material resistance to fracture (critical CTOD) is determined by testing three-point-bend bars following the standard test method, BS5762 [7]. The test specimens are extracted from actual welded pipes. The applied CTOD in the flawed pipe and the material toughness are equated (with safety factors) to generate flaw assessment diagrams of the type shown in Fig. 2. Each curve in the figure separates accept/reject regions for a specified axial stress (strain) level and material toughness. The procedures in API-1104:Appendix A impose practical limits on the maximum crack depth and maximum crack length as indicated by the dashed line in the figure. Cracks detected with a length or depth outside these limits are rejected without additional consideration. Cracks having dimensions within the limits are evaluated following the procedures described in API-1104:Appendix A that incorporate the measured fracture toughness of the material and the applied axial strain. Further simplifications in the evaluation process are achieved by pre-qualifying the welding procedures for two levels of fracture toughness, 0.005 in. and 0.010 in. of CTOD.
In a comprehensive experimental program on the strength of pipeline girth welds, Glover, et al. [19,20] tested a total of 47 pipes to failure in bending. Pipe diameters ranged from 20 in. to 42 in.; wall thicknesses ranged from 0.266 in. to 0.591 in. Part-through, circumferential cracks in the welds were initiated with saw-cut notches on the inner surfaces of the pipes. Fatigue loading was used to extend cracks to the desired sizes. The pipes were then loaded slowly to failure in bending. Tested crack depths ranged from $a/t$ of 0.06 to 0.90; the longest cracks were approximately 35% of the pipe diameter ($L/D = 0.35$).

The few tests conducted at temperatures below $-45^\circ$ C exhibited brittle fracture failures. The majority of tests were conducted at $-45^\circ$ C or warmer. Failure occurred in these tests by ductile tearing which created a through-wall crack and/or by buckling of the compression side prior to ductile crack growth.

The test results were evaluated using the approach defined in PD6493 (without recategorization) to predict the maximum allowable axial strain on the girth weld. Initial comparisons showed a factor of safety against failure between 0.76 and 2.0 for some of the long, shallow flaw cases; safety factors greater than 2.0 were found for short, deep flaws. These values included no provisions for residual tensile strains in the welds. When residual strains equal to one-half the elastic yield strain were included in the analysis, the safety factor in all cases increased to 1.7 or greater. However, the tolerable applied strain was predicted to be less than zero in four cases and extremely large safety factors were obtained in other cases. Recategorization of the flaws as through-thickness cracks, which is required in PD6493 when the net ligament stress exceeds yield, provided very large safety factors but unrealistically low allowable strains.

Two of the 47 tests, Nos. 18 and 19 in [19,20], caused the most concern. The pipe in each case had 36 in. diameter with 0.435 in. wall thickness and was Grade X70 material. The yield and flow stress for the X70 material was 76.9 ksi and 87.0 ksi, respectively. The girth weld metal has a yield and flow stress of 75.8 ksi and 82.0 ksi, respectively. Other parameters for these two tests are summarized in Table 1. Neither of these two pipes qualify for evaluation using API-1104:Appendix A. The CTOD for the weld metal is less than the minimum of 0.005 in. required by Appendix A. Moreover, the weld metal flow stress undermatches the base metal flow stress by $\approx 6\%$. This leads to
Table 1

Parameters for WIC Tests 18 and 19

<table>
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<tr>
<th>Test</th>
<th>$a$ (in.)</th>
<th>$a/t$</th>
<th>$L$ (in.)</th>
<th>CTOD (in.)</th>
<th>$\varepsilon_t$</th>
<th>$\varepsilon_f$</th>
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<tr>
<td>18</td>
<td>0.128</td>
<td>0.3</td>
<td>10.3</td>
<td>0.0039</td>
<td>0.0030</td>
<td>0.0026</td>
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<tr>
<td>19</td>
<td>0.125</td>
<td>0.3</td>
<td>10.8</td>
<td>0.0039</td>
<td>0.0037</td>
<td>0.0028</td>
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(1) CTOD is measured value for weld metal.

(2) $\varepsilon_f$ is the predicted failure strain;

(3) $\varepsilon_t$ is the measured failure strain;

collection of strain in the weld metal above the $\varepsilon_t$ value measured by gages on the pipe wall outside the girth weld. The actual strain at failure reported as $\varepsilon_t$ in Table 1 is thus larger by an unknown amount.

This extensive set of tests pointed to the need for additional study of the long, shallow flaw configuration, particularly the computation of applied CTOD values. Full 3-D finite element models of cracked pipes considered in this study permit computation of more realistic values of the applied CTOD. The modeled flaws are located in the base metal of the pipes. The consideration of a nonlinear, 3-D model containing a surface crack in a girth weld is beyond the scope of this study. A realistic model for a girth weld needs to incorporate material properties in the various heat affected zones. Nevertheless, the fracture driving force for the base metal flaws modeled in this study should closely match the fracture driving force for the weld flaw unless there is significant over/under matching of material strengths in the weld and base metal.

1.3. RELATED PREVIOUS WORK

Since 1980, several experimental and analytical investigations have addressed the elastic-plastic response of surface cracks in pipeline steels. The following sections briefly review the previous studies of relevance to the current work.
1.3.1. Experimental Studies

Ezzat and Erdogan [17] conducted full-scale bending tests on six pipes containing circumferential cracks. The pipes were fabricated from X-60 material to have a 20 in. diameter and 0.34 in. wall thickness. External surface cracks in the base metal of the pipes were introduced with a saw-cut starter notch and fatigued to the desired depth under four-point bending. The fatigue cracks were extended completely through the wall in two of the six pipes. In the remaining four pipes, the fatigue cracks terminated at $a/t$ ratios of 0.55 to 0.77 and $L/D$ ratios of 0.08 to 0.10 (which correspond to $L/t$ ratios of 5.0 to 6.1). The pipes were then loaded slowly in bending at room temperature until failure. Instrumentation recorded the Crack Mouth Opening Displacement (CMOD), load, deflection, and strains.

The Ezzat and Erdogan study produced two major conclusions. First, the fatigue crack growth rate can be predicted using appropriate stress intensity factors in conjunction with standard fatigue data for the material. Second, net ligament yielding did not lead to unstable fracture as had been anticipated. Tearing and necking-tearing behavior occurred in a gradual, stable mode under increased loading. The pipes failed due to extensive plastic deformation in the region of the crack which, in all but one case, was accompanied by buckling on the compression side. For the shallowest crack, $a/t = 0.55$, the net ligament did not rupture and there was no evidence of ductile crack growth at the load that buckled the compression side. CMOD values in the range of 0.020 in. to 0.030 in. were observed at the load required to cause net ligament yielding.

The elastic-plastic behavior of surface cracks in tensile panels fabricated from X-70 material was examined experimentally by Cheng, et al. [10]. The panels, tested at room temperature, had instrumented sections 12 in. long, 4.0 in. wide and 0.63 in. thick. Surface notches were started with saw-cuts and fatigued to various depths. The crack depths tested had $a/t$ ratios in the range of 0.35 to 0.66. In addition to instrumentation to record load, CMOD, and axial extension, the CTOD was measured through a replication technique that made use of a precision-impression material. Sufficient numbers of electrical-resistance strain gages and a brittle lacquer coating were applied to enable the overall strain distribution under large-scale plasticity to be studied.
These experiments demonstrated clearly the complex interaction of yielding in the remaining ligament and yielding at the ends of the crack. Strains in the region immediately behind the crack tip remained comparable to the remotely applied strain field until plastic zones at the ends of the crack reached the free edges of the panel (net section yielding). The remaining ligament then developed extensive necking accompanied by ductile crack growth leading to a stable failure mechanism of ligament rupture. CTOD values measured using the replication technique were in the range of 0.020 in. to 0.060 in. at the initiation of ductile tearing.

As noted in the previous section, an extensive experimental program to investigate the strength of pipeline girth welds was conducted by Glover, et al. [19,20]. In addition to results described previously, this test program also indicated the absence of an unstable fracture event caused by net ligament yielding. The tests of this study further confirmed the observations made in the tests by Ezzat and Erdogan.

1.3.2. Analytical Studies

Three approaches have evolved to analytically model a part-through, circumferential flaw in a pipe under bending loads. The approaches, in order of increasing accuracy, complexity and cost, may be termed: 1) equilibrium models, 2) line-spring models with corrections, and 3) nonlinear, 3-D finite element models. Included with the line-spring models are simple Dugdale or strip-yield type models. The following sections briefly outline the capabilities and limitations of each approach.

1.3.2.1. Equilibrium Models

Models of the type proposed by Zahoor, et al. [39] predict whether a part-through crack grows radially or circumferentially and whether an unstable fracture event is possible from energy considerations. The flaw is idealized by a constant depth, part-through crack of finite length. The model assumes that the pipe cross section is fully plastic under the bending load and that the material toughness is sufficient to reach the plastic moment without tearing. This model does not, therefore, predict a load-CTOD or a load-\(J\) curve. However, the energy released during radial and circumferential virtual crack extensions
imposed on the fully plastic state may be calculated very simply to obtain $J$-integral values for each type of crack growth.

These analyses indicate the propensity for part-through cracks to grow radially for both bending and tension loads, although more strongly for bending than for tension.

### 1.3.2.2. Line-Spring Models

The line-spring model was originally developed by Rice and Levy [32,33] to compute stress intensity factors for part-through surface cracks in flat plates under bending and tension loads. The surface crack is effectively replaced by a through-crack of the same length with a distributed spring, i.e., a line-spring of varying stiffness placed along the through-crack to model the remaining ligament. The line-spring stiffness at each point on the crack front is taken to be equal to the stiffness of the remaining ligament for a single-edge notch (SEN) panel with the same crack depth. Displacements of the crack faces for the through-crack are computed for the remote loading. Displacement compatibility is then enforced between the line-spring and through-crack faces to complete the model.

Parks [24,25] extended the line-spring concept to include the effect of plasticity in the remaining ligament on the stiffness of the line-springs. The two major difficulties with plasticity in a line-spring model are: 1) a nonlinear constitutive law to model interaction of tension and bending on the remaining ligament, and 2) a method to incorporate plasticity at the ends of the crack.

Computation of the linear-elastic displacements along the through-crack (crack face separation and relative rotation) in a pipe under bending and tension presents a formidable problem. For this reason, the line-spring is often incorporated into a 3-node line, finite element for connection to 8 or 9 node isoparametric shell elements. The linear-elastic shell elements model a pipe with a through-thickness circumferential crack to provide the needed crack face displacements. The line-spring elements are inserted along the through-crack to model the remaining ligament of the surface crack. The nodal connections between elements enforce the required displacement compatibility along the crack. In this approach, plasticity is usually confined to the remaining ligament modeled by the line-springs -- plastic deformation at the ends of the crack and
away from the crack plane is ignored. However, effects due to pipe curvature on deformations in the crack plane are properly modeled.

To circumvent the need for a finite element analysis, Erdogan and Delale [16] developed approximate expressions for the linear-elastic displacements along a circumferential through-crack in a pipe under bending and tension. When combined with line-spring stiffnesses from the SEN panel and displacement compatibility equations, a completely analytical line-spring model for the surface cracked pipe is obtained. Unfortunately, the formulation requires the solution of two, coupled integral equations. Plasticity effects are introduced by assuming a fully yielded ligament in pure tension with a Dugdale type strip-yield model applied at the ends of the crack.

King [21] proposed a much simplified form of the line-spring model. Displacement compatibility between the line-spring and the through-crack is enforced only at the center of the crack. This reduces the coupled integral equations to a pair of linear algebraic equations. The line-spring is assumed to be elastic, perfectly plastic. Once the ligament is yielded, a Dugdale type strip plasticity model, modified to reflect pipe bending behavior rather than a tensile panel, is employed to predict additional opening at the center of the crack due to yielding at the crack ends. This model, termed the pipe yielded-ligament model, exhibits generally good agreement in comparisons with test results for short, deep flaws [30].

1.3.2.3. Finite Element Models

Very few elastic-plastic finite element analyses for fully 3-D surface crack geometries have been reported [27,37,38]. Model development and computational costs remain prohibitive even with the availability of graphical modeling software and super-computers. Wilkening, et al. [38] analyzed axial surface flaws in pressure vessels constructed of A533B material. Both CTOD and J-integral values were determined for three flaw sizes. Wellman, et al. [37] analyzed axial surface flaws in pressure vessels fabricated from A517 steel. CTOD values obtained from the 2-D and 3-D finite element analyses were combined with laboratory tests that measured critical CTOD values to predict burst pressures for five vessels. The finite element estimates for burst pressure using the CTOD approach agreed to within 7%, in the worst case, of the
experimentally measured burst pressure. Failure modes of the five vessels ranged from brittle to entirely ductile.

The study of Pick and Worswick [27] addressed the part-through circumferential flaw in a pipe subjected to bending loads. Unfortunately, the 3-D mesh refinement was not detailed sufficiently to allow accurate estimates of CTOD. Moreover, singularity elements were not used at the crack tip. The study focused on the nonlinear moment-rotation response and development of plastic zones. No comparisons of computed and experimental estimates of CTOD were reported.

1.4. OBJECTIVES AND SCOPE

This study employs detailed 3-D finite element models to predict the elastic-plastic response of pipes containing circumferential, part-through surface cracks. Applied CTOD values obtained from the analyses provide a quantitative basis to evaluate the crack driving force component of the flaw assessment procedures described in API-1104:Appendix A. The investigation focuses on the behavior of pipes containing relatively long but shallow surface cracks. The long, shallow crack configuration is the subject of much current interest as a result of recent full-scale tests on pipe girth welds conducted at the Welding Institute of Canada. The tests indicated that flaw assessment procedures such as BSI-PD6493, and possibly API-1104:Appendix A, were unconservative for the evaluation of long, shallow flaws.

The present study consists of two parts. In the first part, 3-D nonlinear finite element analyses were performed for three crack configurations ranging in size from a short, deep crack with $a/t = 0.5$ and $L/t = 4$ to long, shallow cracks with $a/t = 0.125, 0.25$ and $L/D = 0.40$. The models were slowly loaded in bending. The nonlinear finite element solutions established relationships between applied moment, average wall stress, average wall strain, CTOD, and plastic zone development. In the second part of the study, a full-scale test of a pipe containing a long, shallow flaw was conducted to provide data for verification of the finite element models used in the analytical study.

The finite element models are constructed of parabolic (16, 18, and 20 node) isoparametric elements. Degenerate 20-node elements are placed along the crack front to enforce a $1/r$ strain singularity characteristic of a crack tip in
an elastic-perfectly plastic material. The models contain approximately 2000 nodes and 375 isoparametric elements. Finite element CTOD values are obtained directly from displacements of collapsed nodes on the singularity elements using the 90° intercept method. Incremental plasticity theory is adopted to describe the material response. Finite element estimates for the CTOD are compared to those predicted by PD6493, a strip-yield model and a line-spring model.

For all finite element models of this study, the crack resides in the base metal of the pipe. Realistic modeling of a surface crack located in a weldment is outside the scope of this investigation. Nevertheless, the numerical results should adequately characterize the situation of a crack located in the weld when the base metal and weld metal have similar yield strengths and strain hardening properties.

Under contract to API, the Welding Institute of Canada performed the test to verify the finite element solutions. The tested pipe, fabricated from X-60 material, has a 36 in. diameter and 0.9 in. wall thickness. An external surface crack of dimensions $a/t=0.13$ and $L/D=0.38$ was introduced. The pipe was loaded to failure in bending following the same procedures developed in the earlier series of tests on pipeline girth welds. A 3-D finite element analysis of the tested pipe was performed using the techniques described above. Instrumentation placed on the tested pipe, including near crack and remote strain gages and CMOD gages, recorded data for comparison with finite element results.

The finite element results are also used to study other aspects of the API-1104:Appendix A procedures. The minimum levels of material toughness (0.005 in. and 0.010 in. CTOD) required for application of Appendix A are examined relative to applied CTOD values found necessary to produce net ligament yielding and shear band formation. The discussion also addresses the issues of material toughness values obtained using deeply cracked test specimens for shallow crack assessments and potential benefits of overmatching by the weld metal.

The remainder of this report is divided into four additional chapters. Chapter 2 describes the details of the finite element models and the solution procedures used for the analytical study of the three crack configurations and
for analysis of the tested pipe. Numerical results for the three crack configurations considered in the analytical study are presented in Chapter 3. The crack driving force component of the API-1104:Appendix A procedures are discussed relative to the finite element results. Chapter 4 describes the full-scale pipe test and the corresponding finite element analysis. Comparisons between experimentally measured and finite element results for CMOD, load, and axial strains are presented. A summary of the work performed in this study and the major conclusions are given in Chapter 5.
 CHAPTER 2

FINITE ELEMENT MODELS

2.1. GENERAL

This chapter briefly describes the modeling and solution procedures adopted to perform the 3-D nonlinear analyses. The same modeling and solution procedures were followed for analyses in the parametric study and for analysis of the experimentally tested pipe.

Figure 3 shows the section of pipe represented by the finite element models. In all analyses, the outside diameter of the pipe is 36 in. The part-through crack is located on the outside surface of the pipe. Figure 4 illustrates the parameters that describe the crack size, positions along the crack front, and locations along the inside wall surface.

The numerical computations were performed using the POLO-FINITE [14,15] software operating on a Harris-1000 computer.

2.2. ELEMENT GRIDS, CONSTRAINTS AND LOADING

Symmetry of the loading and the crack geometry permits use of a one-quarter finite element model of the pipe. The mesh includes a 180° arc of the cross section and extends one diameter in the longitudinal direction to enable development of plastic zones in the crack region without restriction from loads and constraints applied on the remote end.

The models are loaded in bending about the X-axis by a force-couple applied on the remote end. The bending moment is oriented to produce maximum tensile stress at the center of the crack. Multi-point constraints are specified for nodes on the remote end to enforce the plane sections assumption of beam theory. The multi-point constraints also eliminate stress concentrations from the two nodal loads applied to produce the couple. Appropriate constraints defined along the boundaries enforce the symmetry conditions.
Figure 5 shows the typical element grid developed to model the pipe. The element meshes contain approximately 2,000 nodes (6,000 degrees of freedom) and 375, 3-D isoparametric elements; the exact number of nodes and elements varies with the crack length and depth. A mixture of 20, 18, and 16-node solid elements are employed with 20-node elements concentrated in the vicinity of the crack. Several 15-node wedge elements are used to simplify transitions between fine and coarse regions of the mesh. The use of 18 and 16-node paralinear elements remote from the crack region reduces the number of nodes in the meshes.

Concentric rings of polyagonal-shaped elements (four elements per ring) model the sharp crack tip as illustrated in Fig. 5b. The inner-most ring contains four, 20-node elements each collapsed into a wedge with side nodes retained in the mid-point position. The collapsed (degenerate) elements impose a 1/r singularity on strain components in principal normal planes along the crack front. Strain components tangent to the crack front remain non-singular. Initially coincident nodes of the degenerate elements along the crack front are left uncoupled to permit blunting. For each crack configuration, four elements are defined along the half length, $a$, of the crack.

The finite element models are generated with the PATRAN-II software system [26]. The construction of each model proceeds in a series of steps. A 3-D finite element model for a flat panel containing a semi-circular surface crack is constructed within PATRAN-II. Nodal renumbering also is performed to minimize the bandwidth and profile. Coordinates of nodes in the crack region are modified using an elliptical transformation [23]. This is accomplished external to PATRAN-II via the neutral file system. Coordinates for the flat panel model are then mapped onto a cylindrical surface to generate nodal coordinates for the pipe model. Additional processing of the nodal coordinates is necessary to reposition mid-side nodes of the elements at the geometric center of the element edges (the elliptic and cylindrical mappings are nonlinear and thus distort positions of mid-side nodes relative to adjacent corner nodes). The nodal coordinates for the pipe model are finally passed to PATRAN-II via the neutral file to establish the model for post-processing of the finite element results.
2.3. PLASTICITY MODELING

Plasticity is modeled using the incremental (path-dependent, $J_2$) theory with a Huber-von Mises yield surface, associated flow rule, and isotropic hardening. Piecewise linear representations are used for the uniaxial, tensile stress-strain curves.

Figure 6 shows the stress-strain curve for an X-70 material which is used in the parametric study described in the next chapter. The proportional limit for the material is 85.5 ksi. The saturation (flow) stress is 92.5 ksi. A yield stress of 88 ksi is defined to normalize stress values in plots of numerical results.

The elastic predictor-radial return method with sub-incrementation is adopted to integrate the plasticity rate equations [13]. Use of a path-independent updating strategy within each load step prevented spurious numerical unloading of the material. The consistent tangent operator for the elastic predictor-radial return algorithm is used in forming the global stiffness of the structure. This computational procedure maintains optimal convergence rates for the Newton-type solution.

2.4. SOLUTION PROCEDURE

The finite element formulation yields a set of nonlinear equilibrium equations in terms of the nodal displacements. The equations are derived by applying the principle of virtual displacements [5]. The conventional linear strain-displacement relations are employed in the present analyses; nonlinear response is thus limited to the material stress-strain behavior. The analyses neglect ovaling of the cross-section. However, the effects of ovaling for a bending, rather than pressure, load are expected to be negligible. The Pick and Worswick study [27] included geometric nonlinearity but showed essentially no effects due to ovaling of the cross-section for bending loads.

The nonlinear equilibrium equations are solved using an incremental-iterative Newton algorithm. The bending moment is increased to a value exceeding the yield moment in 15-20 variable size increments (load steps). Corrective iterations to restore equilibrium are performed at constant applied moment for each load increment until the Euclidean norm of the residual nodal loads is reduced to less than 0.1% of the Euclidean norm of the total nodal
loads. The structure tangent stiffness is updated and re-triangulated prior to iterations $2 \to n$ of the load step. Stiffness updates at the beginning of a load step (iteration 1) do not improve the rate of convergence. Typically, three iterations are needed in each step to achieve a converged solution.

Preliminary computations demonstrated the need for full (3x3x3) Gauss quadrature to prevent development of zero-energy modes in the collapsed tip elements. Full integration is also necessary in distorted elements away from the crack tip to correctly reproduce constant stress states under simple axial loading.

The analyses were performed on a Harris-1000 computer. For 20 load steps, with a total of 40 stiffness updates, approximately 60 hours of CPU time were required.

2.5. DEFINITION OF FINITE ELEMENT CTOD VALUES

The use of collapsed 20-node elements along the crack front provides a convenient means to extract CTOD values from displacements of the initially coincident nodes. At a point on the crack front, the finite element CTOD value is defined as the separation of the crack faces at the location where two rays, drawn at right angles to each other from the current crack tip, intercept the crack faces (see Fig. 7). This technique to define CTOD was suggested by Rice [31] and is commonly referred to as the $90^\circ$ intercept method.
CHAPTER 3

PARAMETRIC STUDY OF SURFACE CRACKS

3.1. GENERAL

This chapter describes a limited parametric study of surface crack sizes on the nonlinear fracture response of pipes in bending. The expense of 3-D, nonlinear finite element analyses limited consideration to two long, shallow flaws and one deep, short flaw. The finite element results are employed to evaluate the crack driving force component of the flaw assessment procedures described in API-1104:Appendix A with emphasis on long, shallow flaws.

Results from linear analyses of each configuration are described briefly to verify the adequacy of the finite element mesh. Stress-intensity factors, $K_I$, along the crack front are compared to flat plate solutions.

The nonlinear behavior of each cracked pipe is presented in terms of the overall load-deflection response, the formation of plastic zones, axial strain distributions on the inside and outside surfaces near the crack, and the CMOD-CTOD values along the crack front. Finite element results for CTOD are compared to values given by a strip-yield model, a line-spring model, and by the PD6493 design curve.

The assessment of Appendix A focuses on the load levels needed produce applied CTOD values of 0.005 in. and 0.010 in. Girth welds must be pre-qualified for these minimum levels of toughness for application of Appendix A. The required loading levels are expressed in terms of the bending moment, nominal wall stress, and nominal wall strain. Corresponding values of nominal wall strain computed using the Appendix A procedures are compared to finite element results adjusted to include a residual strain contribution. The applied CTOD values at which Net Ligament Yielding (NLY) occurs are also established from the finite element results. The values are compared to the 0.005 in. and 0.010 in. toughnesses to indicate the amount of ductility implied by the Appendix A requirements.
The chapter concludes with a qualitative discussion of two other factors which enter into flaw assessments. These factors are the relatively severe conditions in the three-point bend test which leads to conservative values of critical CTOD and the potential benefits of weld metal overmatching for reducing the applied CTOD values.

3.2. CRACK CONFIGURATIONS

Finite element analyses were performed for three surface crack sizes summarized in the table shown on Fig. 4. The pipe outside diameter and wall thickness are fixed at 36 in. and 0.8 in., respectively.

The crack configurations are denoted A, B, and C. Points corresponding to these crack sizes are indicated on the qualitative flaw assessment diagram in Fig. 2. Configuration A represents the maximum length shallow flaw \((a/t=0.25, L/D = 0.4)\) permitted in API-1104:Appendix A. Case B has the same crack length \((L/D = 0.4)\) but one-half the depth \((a/t=0.125)\). The analysis for Case B examined the sensitivity of CTOD to crack depth for maximum length flaws. Configuration C represents the maximum length deep flaw \((a/t = 0.5, L/t = 4)\) permitted in API-1104:Appendix A. The analysis for this case determined the relative severity, in terms of the applied CTOD, between the maximum length shallow flaw and maximum length deep flaw.

3.3. FINITE ELEMENT RESULTS

3.3.1. Linear Analyses

A linear-elastic analysis was performed for each crack configuration to verify the adequacy of the element mesh. Mid-side nodes of the singularity elements along the crack front were displaced to the quarter-point position for these analyses. Coincident nodes of the collapsed singularity elements were constrained to have zero displacement normal to the crack plane. These modifications enforced the \(1/\sqrt{r}\) singularity at the tip for strain components in the principal normal planes along the crack front.

Stress-intensity factors were computed by substituting nodal displacements normal to the crack face into the asymptotic solution. Shih, et al. [34] specialized the procedure for use with quarter-point elements. The stress-intensity
The factor is given by

\[ K_I = \frac{E}{2(1 - \nu^2)} \sqrt{\pi/2L} \left( 4V_B - V_C \right) \]  

where \( E \) is Young’s modulus, \( \nu \) is Poisson’s ratio, \( L \) is the dimension of the crack tip element, and \( V_B, V_C \) are the opening displacement of the quarter-point node and adjacent corner node, respectively. This expression is applicable for the plane strain condition which prevails over the crack front except at the free surface.

Table 2

Comparison of Linear-Elastic \( K_I \) for Finite Element Analyses with Flat Plate Solutions

<table>
<thead>
<tr>
<th>Case A ( (a/l = 0.25, \quad L/D = 0.4) )</th>
<th>Case B ( (a/l = 0.125, \quad L/D = 0.4) )</th>
<th>Case C ( (a/l = 0.5, \quad L/l = 4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>( F_{fe} )</td>
<td>( F_{ec} )</td>
</tr>
<tr>
<td>90°</td>
<td>1.48</td>
<td>1.50</td>
</tr>
<tr>
<td>75°</td>
<td>1.47</td>
<td>1.47</td>
</tr>
<tr>
<td>60°</td>
<td>1.37</td>
<td>1.38</td>
</tr>
<tr>
<td>45°</td>
<td>1.34</td>
<td>1.29</td>
</tr>
<tr>
<td>30°</td>
<td>1.18</td>
<td>1.20</td>
</tr>
<tr>
<td>15°</td>
<td>1.24</td>
<td>1.14</td>
</tr>
</tbody>
</table>

where,

\[ K_I = F \sigma \sqrt{\pi a} \]

\( F_{fe} \) = Finite element this study

\( F_{ec} \) = SEN tensile panel

\( F_{NR} \) = Newman-Raju [23,28]

Table 2 compares the stress-intensity factors computed using the above procedure with solutions for flat plates. For Cases A and B, the large crack lengths preclude comparisons with the extensive results given by Newman and Raju [23,28]. Instead, the 3-D stress-intensity factors are compared to the values for a single edge notch (SEN) panel. The crack length for the SEN solution is taken as the actual depth of the surface crack corresponding to the location on the crack front. The 3-D finite element and SEN values agree very well.
except near the ends of the crack. The stress-intensity factor attains a maximum value at the center of the crack ($\phi = 90^\circ$) and decreases near the free surface. The exact variation of stress-intensity factor near the free surface is complicated by a boundary layer effect [28]. No attempt is made in this study to accurately model the free-surface effects at the ends of the crack.

For Case C (the short, deep flaw), Newman and Raju's results are applicable. Comparisons of the stress-intensity factors are also given in Table 2. The values are in close agreement except near the free surface. In this case, the last point of comparison on the crack front ($\phi = 15^\circ$) is considerably closer to the free surface than points located at the same $\phi$ angle for the much longer cracks of cases A and B.

3.3.2. Nonlinear Load-Deflection Response

The computed load-deflection relationships are given in Figs. 8-10. The bending moment characterizes the applied load and is normalized by the moment required to yield the extreme fiber of the uncracked pipe, $M_y$. The applied moment is plotted against the average (nominal) axial strain in the pipe wall at the remote end. This strain value is normalized by the material yield strain, $\epsilon_y$. The normalized wall strain is also equal to the pipe curvature normalized by the curvature at first yield of the extreme fiber ($t/R \ll 1$ so that variations of stress-strain over the wall thickness are negligible). Normalized values of stress, strain, and moment are adopted to permit use of the numerical results for materials that have different yield strengths but with strain-hardening characteristics similar to those of X-70 shown in Fig. 6.

Each figure shows that the computed response for the cracked and uncracked pipes exhibit negligible differences. This is expected since the largest crack (Case A) reduces the cross-sectional area by 3.3% and the applicable moment of inertia by 6.5%.

The load-deflection response for an uncracked pipe was obtained using a nonlinear filament model and the uniaxial stress-strain curve shown in Fig. 6. A $90^\circ$ section of the wall was divided into 200 triangular shaped elements. A sequence of increasing values for the curvature was imposed on the cross-section. For a specified value of curvature, the strain in each triangle was found using the plane-sections approximation of simple bending. The
corresponding stress in each triangle was found from the uniaxial stress-strain curve. The stress at the centroid of each triangle, multiplied by the area of the triangle, determined the force. The moments of all such forces about the neutral axis were summed to determine the resultant bending moment corresponding to the applied curvature.

For reference, the CTOD values at the center of the crack are shown in () at each load step on Figs. 8-10.

3.3.3. Development of Plastic Zones

Useful insight is gained into the overall nonlinear response of a cracked pipe by studying the formation and growth of plastic zones. Moreover, the changing rates at which CTOD increases under applied loading are related directly to the extent and location of plastic deformation in the pipe.

Plastic zones form at each location along the tip of the surface crack and propagate diagonally toward the inside surface of the pipe wall as illustrated in Fig. 11. The plastic zones eventually reach the inside surface of the pipe wall, a condition termed Net Ligament Yield (NLY). The applied loading required to cause NLY is determined in the finite element analysis by tracking the plastic zones as they extend diagonally from each point along the crack front through the remaining wall thickness.

If a crack has sufficient depth, axial strain concentrates in the diagonal plastic zones producing shear bands which appear as a spike in the axial strain distribution on the inside surface of the pipe wall at a small distance above the crack plane. This phenomenon is illustrated in Fig. 11b. The axial strain distributions along the inside wall at the center of the crack ($\theta = 0$) for Case A are shown in Fig. 12a. Axial strain distributions are given that correspond to three values of CTOD (0.005 in., 0.010 in., and 0.038 in.). The strain spike is located approximately 1.0 in. above the crack plane. The small spike that occurs at a CTOD of 0.004-0.005 in. corresponds to the plastic zone reaching the inside surface. Axial strain distributions at other locations along the crack front ($\theta > 0$) are very similar.

For very shallow cracks, the through-thickness shear bands do not form until load levels well in excess of those at Net Ligament Yield are reached. In
such cases, the wall stress on the net section differs little from the average wall stress; complete yielding through the wall thickness occurs almost simultaneously at the net ligament and elsewhere remote from the crack. This prevents the formation of shear bands as demonstrated by the axial strain distributions for Case B given in Fig. 12b.

Plastic zones and shear bands in the circumferential direction also tend to develop at the two ends of the crack. However, extension of these secondary shear bands is blunted by the linearly decreasing stress field from overall bending of the pipe.

Continued loading of the pipe beyond $M_y$ eventually creates a pattern of plastic deformation corresponding to that for an uncracked pipe in simple bending. This pattern of deformation is termed Gross Section Yielding (GSY). An important result of GSY is that material in the pipe wall outside the diagonal shear bands also yields which restores a more uniform stiffness variation in the crack region. This influences the rate of increase of CTOD as discussed in next section.

3.3.4. CTOD Values

The finite element estimates for CTOD at the center of each crack are plotted as a function of average wall stress, average wall strain, and applied moment in Figs. 13-15. The average wall stress is normalized by the yield stress of the material. Wall strain and applied moment are normalized as described previously. These figures clearly demonstrate that the deep flaw, Case C, represents the most severe of the three crack configurations analyzed in terms of applied CTOD values.

Figures 13-15 reveal that the very shallow flaw (Case B with $a/t = 0.125$) does not attain a CTOD value in the 0.005 in. range until the applied moment approaches $1.3 M_y$. The corresponding wall strain exceeds $2.5 \epsilon_y$. For this flaw depth (and shallower flaws), shear bands do not form through the wall thickness; consequently, there is no significant change in the response for further loading after the remaining ligament yields. In these cases, the elastic-plastic response is more properly characterized as Gross Section Yielding without prior Net Ligament Yielding since the two conditions develop almost simultaneously. Pipes containing shallow flaws with this type of nonlinear response are more
likely to fail by plastic collapse or buckling than by brittle or ductile fracture.

For Cases A and C, the applied moment must exceed \( M_y \) for the CTOD to attain a value of 0.010 in. To attain the 0.005 in. range of CTOD, the applied moment must reach 0.94 \( M_y \) and 0.85 \( M_y \) for Cases A and C, respectively. For both Cases A and C, distinct through-thickness shear bands form at Net Ligament Yield. This leads to the increasingly larger values of CTOD for small increases in loading as can be seen clearly in Fig. 14. Table 3 summarizes the load levels required to attain 0.005 in. and 0.010 in. of CTOD for each configuration.

### Table 3

<table>
<thead>
<tr>
<th>CTOD (in.)</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTOD (in.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma / \sigma_y )</td>
<td>0.94</td>
<td>0.96</td>
<td>( &gt; 1.0 )</td>
</tr>
<tr>
<td>( M / M_y )</td>
<td>0.94</td>
<td>1.05</td>
<td>1.2</td>
</tr>
<tr>
<td>( \epsilon / \epsilon_y )</td>
<td>0.94</td>
<td>1.08</td>
<td>( &gt; 2.0 )</td>
</tr>
</tbody>
</table>

The effects of GSY on reducing the rate at which CTOD increases is observed in Fig. 14. The CTOD levels at which GSY begins to develop for Cases A and C are 0.024 in. and 0.012 in., respectively. However, the pipe may fail before these load levels can be reached unless the material has comparable levels of toughness. Plastic collapse or buckling may also occur prior to attainment of the corresponding magnitudes of wall strain.

### 3.3.5. CTOD Variation Along Crack Front

The variation of CTOD along the crack front as a function of applied moment is shown in Figs. 16-18. The CTOD attains a maximum value at the center of the crack in each case. The minimum value occurs at the ends of the crack. The largest differences between the center point and end point CTOD values occur for the long, shallow cracks, Cases A and B. For the deep crack, the differences between CTOD values at the center point and end point are not
as large since the applied stress (strain) is essentially constant over the short crack length.

3.3.6. CTOD Variation With CMOD

The relationship between CTOD and the Crack Mouth Opening Displacement (CMOD) is shown in Figs. 19-21. CMOD is an important parameter since it can be measured in an experiment. The CTOD increases parabolically with CMOD at low loads when the plastic zones are contained within the wall. The crack faces are highly curved during this stage of loading. When the NLY condition is approached, a hinge mechanism forms in the wall after which the CTOD increases linearly with CMOD. The crack faces rotate relative to each other which contributes a component to CMOD in addition to the CTOD component. The results show that after NLY the CTOD is $\approx 1/2$ of the CMOD for each crack. Similar relationships between CTOD and CMOD are observed for standard test specimens including the three-point bend bar and the compact-tension specimen.

3.4. CTOD ESTIMATES FROM OTHER MODELS

Applied CTOD values for pipes with surface cracks may be estimated using simple models that ignore the geometric complexity of the crack shape, pipe curvature, and strain hardening of the material. Three such models are used to estimate applied CTOD values for comparison with the finite element values. They are: 1) the design curve in PD6493 used as a model to predict CTOD, 2) a collapse modified strip-yield model suggested by Garwood [18], and 3) a simplified line-spring model developed by King [21].

3.4.1. PD6493 and Strip-Yield Models

For the PD6493 procedure and the strip-yield model, the surface crack is converted to an equivalent through-thickness crack of half-length $\bar{a}$ in an infinite plate loaded in tension. The conversion requires that the stress-intensity factor for the actual crack and equivalent crack be identical. The charts provided in PD6493 are used to estimate equivalent half-crack lengths of 0.44 in., 0.16 in., and 0.72 in. for Cases A, B, and C respectively.
CTOD estimates for the PD6493 model are given by:

\[
\delta = \left( \frac{\sigma}{\sigma_y} \right)^2 \frac{2\pi \sigma_y \bar{a}}{E}; \quad \text{for} \quad \frac{\sigma}{\sigma_y} \leq 0.5
\] (2)

\[
\delta = \left( \frac{\sigma}{\sigma_y} - 0.25 \right) \frac{2\pi \sigma_y \bar{a}}{E}; \quad \text{for} \quad \frac{\sigma}{\sigma_y} > 0.5
\] (3)

For the strip-yield model, the values are given by:

\[
\delta = \frac{8\sigma^2 \bar{a}}{\pi E \sigma_y} \ln \left[ \sec \left( \frac{\pi \sigma}{2 \sigma_y} \right) \right]
\] (4)

where \( E \) is Young's modulus, \( \sigma_y \) is the yield stress, and \( \sigma \) is the average wall stress computed from the bending formula for the applied moment. A yield stress of 88 ksi is defined for use in the above equations. This value represents the average of the proportional limit (85.5 ksi) and the saturation stress (92.3 ksi) for the X-70 material.

3.4.2. Simplified Line-Spring Model

The simplified form of the line-spring model proposed by King [21] is used here. The model enforces displacement compatibility between the line-spring and the through-crack only at the center of the crack. This reduces the coupled integral equations that appear in the conventional line-spring model to a pair of linear algebraic equations. A correction factor is included for the effect of pipe curvature on displacements of the through-crack. The line-spring is assumed to be elastic, perfectly plastic. A yield (flow) stress of 92.3 ksi is used in present computations which corresponds to the saturation stress of the material. Once the ligament is yielded, a strip-yield model is adopted to predict additional opening at the center of the crack due to yielding at the ends of the crack.

3.4.3. Comparison of Estimates for CTOD

CTOD estimates computed using the three simple models and the finite element results are compared in Figs. 22-24. The strip-yield, line-spring, and finite element results agree well in each case for wall stresses less than 0.5\( \sigma_y \). In this range of applied stress, the design curve in PD6493 estimates a CTOD approximately 2.0-2.5 times larger than the other models. This reflects the
safety factor incorporated in the design curve at these stress levels -- results for
the strip-yield model, the line spring model and the finite element results have
no safety factor included.

At stress levels between 0.8–0.9 $\sigma_y$, the strip-yield, line-spring and finite
element models predict rapidly increasing CTOD values at the the onset of
NLY. The design curve in PD6493 simply increases the CTOD linearly for
$\sigma > 0.5\sigma_y$. The line-spring model exhibits somewhat better agreement with
the strip-yield and finite element results for the short, deep flaw where end
effects of the surface crack contribute significantly to the response. For long,
shallow flaws, the line-spring model overestimates the additional contribution to
CTOD from yielding at the ends of the surface crack. The incorrect response is
caused by the very long crack length used in a simple Dugdale model to incor­
porate the end effects. Just prior to NLY, the line-spring underestimates the
CTOD in comparison with the finite element values. At NLY, the line-spring
results exhibit a sharp break indicating the instantaneous yielding of the com­
plete ligament. The line-spring could be improved by using a through-thickness
plasticity correction prior to NLY which would increase the CTOD slightly and
also provide a more gradual transition from a fully elastic net ligament to a
fully yielded net ligament.

In each case, the finite element estimates of CTOD represent the lower
bound of all results once the NLY condition is approached. This is not unex­
pected since the 3-D finite element models reflect the influence of progressive
yielding through the thickness, strain hardening of the material, constraint
effects on deformation parallel to the crack front, and pipe curvature. Each of
these factors acting alone decreases the CTOD relative the values predicted by
the simpler models that do not account for such effects.

3.4.4. The PD6493 Design Curve

Figure 25 shows the ratios of applied CTOD values predicted by the
PD6493 design curve to the values computed in the finite element analyses.
For low levels of loading, $\sigma < 0.5\sigma_y$, the ratio is approximately 2.0 for each
 crack configuration. In this loading range, the response is adequately predicted
by the stress-intensity factor corrected for small-scale yielding at the crack tip.
The PD6493 curve is constructed from such a model. The 2.0 factor is
specifically built into the PD6493 curve to provide a safety factor on the crack force driving component of flaw assessments. However, as the applied stress increases and approaches \( \sigma_y \), the ratio gradually decreases and then finally falls below 1.0 for Cases A and C. For the very shallow crack, Case B, the ratio actually rises to 3.0 with increased stress then sharply falls to below 1.0.

This highly variable ratio follows from the simplified model for crack driving force. The stress-intensity factor type model cannot reflect the complex nonlinear response for a wide range of surface cracks subjected to mixed tension and bending. For this reason, recommendations have been made \([4,11]\) to limit application of the PD6493 design curve to stress levels less than \(0.85 \sigma_y\).

3.5. ASSESSMENT OF API-1104:APPENDIX A

3.5.1. General Observations

The finite element estimates for CTOD shown in Figs. 13-15 lead to three immediate observations. First, the maximum length deep flaw (Case C) and the maximum length shallow flaw (Case A) permitted in API-1104:Appendix A have comparable values of CTOD under equivalent loading. The deep flaw consistently exhibits somewhat larger values and represents the more severe configuration. If a flaw assessment procedure is consistent, then Cases A and C both should be accepted or rejected on the basis of applied CTOD values.

The second observation is that detection of the flaw depth is especially important for long flaws. A comparison of the results for Cases A and B clearly shows the influence of crack depth on CTOD values. CTOD values for Case A are 3-4 times greater than those for Case B at applied moments less than the yield moment; the difference becomes even larger once the yield moment is exceeded.

The third observation concerns the magnitudes of loading required to attain CTOD values of 0.005 in. and 0.010 in., as shown in Table 3. These values suggest that pipes containing defects characterized by Cases A, B, and C should not fail by fracture prior to NLY if the material has a minimum toughness in the range of 0.005 in CTOD. When the NLY condition can be developed without fracture, other failure mechanisms including plastic collapse and/or buckling, in addition to fracture, also require investigation.
3.5.2. Estimates of Applied CTOD

The finite element values for CTOD may be used to evaluate the procedures in API-1104:Appendix A that estimate the applied CTOD, specifically Figure A5. Figure A5 relates CTOD to applied axial strain and flaw depth and is reproduced here as Fig. 26. To conduct the comparison, Figure A5 is used as a model to predict CTOD as follows: 1) enter Figure A5 with a known flaw depth, for example 0.2 in., 2) proceed horizontally until the 0.005 in. CTOD curve is reached, 3) the corresponding axial strain is read from the abscissa, in this case approximately 0.002. Figure A5 indicates a CTOD value of 0.005 in. for a 0.2 in. deep crack for an applied axial strain of 0.002. Similarly, the CTOD is 0.010 in. for a strain of 0.0053. For axial strains between these points (0.002 and 0.0053), the CTOD is linearly interpolated between the 0.005 in. and 0.010 in. curves given on the figure.

The curves drawn on Figure A5 reflect increased CTOD values for residual strains of 0.002 in the girth welds. The finite element CTOD values for an applied axial strain must be adjusted upwards by an additional CTOD due only to residual strain (stress). A CTOD of 0.001 in. caused by residual strain at stress levels below $\sigma_y$ is realistic for this material and the crack depths considered. The 0.001 in. value is obtained by assuming a residual stress equal to $\sigma_y$ that is resisted in a linear-elastic manner. Simple conversions from $K_I$ to the $J$-integral, to the CTOD provide the 0.001 in. CTOD due to residual strain (stress). Experimental data recently obtained at the National Bureau of Standards [29] confirms that this level of CTOD due to residual strains in welds may actually be an upper bound and that values less than 0.001 in. are probable.

Table 4 summarizes the data for the long, shallow flaw (Case A) computed using the above procedure. Because the minimum toughness allowed in Appendix A is 0.005 in. CTOD, a comparison is not possible for applied strains less than 0.002, i.e., a minimum toughness curve of 0.002 in. or 0.004 in. CTOD is required for the calculations. Nevertheless, Table 4 indicates that Fig. A5 estimates CTOD values greater than 1.5 times the finite element values provided the applied strain (due to external loading) does not exceed $0.85 - 0.9 \epsilon_y$. If the finite element values are not adjusted for residual strains, the ratios are $\geq 2$ for applied strains less than $0.85 - 0.9 \epsilon_y$. This comparison
Table 4

FEA and Appendix A CTOD Values for Case A
\((a/t=0.25, L/D=0.4)\)

<table>
<thead>
<tr>
<th>(\varepsilon_a) (1)</th>
<th>(\varepsilon_a/\varepsilon_y)</th>
<th>CTOD-1104 (2)</th>
<th>CTOD-FEA (3)</th>
<th>Ratio (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0020</td>
<td>0.71</td>
<td>0.0052</td>
<td>0.008</td>
<td>1.7</td>
</tr>
<tr>
<td>0.0025</td>
<td>0.88</td>
<td>0.0060</td>
<td>0.004</td>
<td>1.5</td>
</tr>
<tr>
<td>0.0030</td>
<td>1.05</td>
<td>0.0067</td>
<td>0.008</td>
<td>0.8</td>
</tr>
</tbody>
</table>

(1) Axial strain from external loading;
(2) CTOD from API-1104, Fig. A5;
(3) CTOD from FEA plus 0.001 in. for residual strain;
(4) CTOD from Fig. A5 divided by CTOD from FEA.

shows that a variable degree of conservatism on the crack driving force exists in Fig. A5 for this crack size. If the applied strain is allowed to exceed \(\varepsilon_y\), the crack driving force is underestimated. The overall safety factor against fracture, for which driving force is but one component, is reduced accordingly.

Table 5

FEA and Appendix A CTOD Values for Case C
\((a/t=0.5, L/t=4)\)

<table>
<thead>
<tr>
<th>(\varepsilon_a) (1)</th>
<th>(\varepsilon_a/\varepsilon_y)</th>
<th>CTOD - 1104 (2)</th>
<th>CTOD - FEA (3)</th>
<th>Ratio (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.175</td>
<td>0.0063</td>
<td>0.0015</td>
<td>4.2</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.351</td>
<td>0.0078</td>
<td>0.002</td>
<td>3.9</td>
</tr>
<tr>
<td>0.0015</td>
<td>0.526</td>
<td>0.0092</td>
<td>0.003</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table 5 presents a similar comparison for the long, deep flaw (Case C). The ratios for this case exceed 3.0 for the limit of 0.0015 applied strain imposed by Figure A5. For this flaw, it appears that Fig. A5 maybe overly conservative in limiting the applied strain to just 0.0015.
Similar comparisons for Case B are not possible using Figure A5. A 0.001 in. or 0.002 in. CTOD curve is needed to perform the evaluation. Toughness values this low are not permitted for application of Appendix A and are thus not included on the figure. Moreover, the finite element results demonstrate that flaws with $a/t < 0.125$ exhibit a GSY response without formation of through-thickness shear bands. Axial strains exceeding $\epsilon_y$ may be safely applied in such situations without producing CTOD values larger than 0.005 in.

The current version of Fig. A5 uses the actual crack depth and applied strain for the axes. In future versions of this figure, a switch to non-dimensional axes ($a/t$, $\epsilon/\epsilon_y$) may eliminate much of the variability in the assessment of short and long flaws.

3.5.3. Influence of Testing Procedure and Overmatching

Two additional factors should be considered in an overall assessment of Appendix A. These are the testing procedure to establish fracture toughness of the weld and overmatching/undermatching of yield strengths between the base metal and weld metal. Each of these factors is discussed briefly below.

API-1104:Appendix A requires that the critical CTOD values be obtained using the three-point bend specimen according to the procedures defined in S5762:1979. Use of the preferred cross-section geometry (thickness B by $\text{pth} B$) is required and the crack depth must lie in the range of 0.45-0.55 of $t$. Critical CTOD values measured for an $a/t = 0.5$ are often much lower (1/2 less) than values measured for test specimens having an $a/t < 0.25$. Critical TOD values are also found to increase when the remaining ligament is loaded tension or combined tension-bending.

Sources of the increased toughness values measured for shallow crack specimens and for tension loaded specimens are not yet fully understood. However, the spread of plasticity beyond the simple hinge region and the subsequent loss of through-thickness constraint are believed to be major factors. Nevertheless, fracture assessments of shallow flaws in girth welds, which are ded primarily in tension, using toughness values from standard three-point specimens will substantially increase the overall safety factors above those applied in Appendix A.
The second factor to consider is overmatching/undermatching of yield strengths between the weld metal and base metal. Overmatching is common practice in girth welds, particularly for line pipe steels that have lower yield strengths. Current research at the National Bureau of Standards is addressing the influence of overmatching/undermatching on applied CTOD values for cracks in the weld metal. Early experimental and computational studies indicate that as little as 20% overmatching significantly reduces the applied CTOD values for small (shallow) flaws of the type found in girth welds. For large flaws there is essentially no benefit gained by overmatching— from a fracture mechanics viewpoint.

Overmatching reduces the effects of a small crack by promoting development of the Gross Section Yield condition with a corresponding more uniform distribution of applied strain. Undermatching increases the applied CTOD once there is significant plasticity. These results suggest that overmatching should be used whenever possible and that for small cracks, a beneficial reduction of applied CTOD values can be expected. The net effect is to further increase safety factors in the flaw assessment procedures.

3.6. CONCLUDING REMARKS

This chapter presented the results of 3-D nonlinear finite element analyses for three sizes of surface cracks in pipes subjected to bending loads. The analyses provided data to evaluate the crack driving force component of the flaw assessment procedures described in API-1104: Appendix A.

The relationships between increasing values of CTOD and wall stress, wall strain, and applied moment were established for three crack sizes. The extent of plastic deformation in each model was related to the corresponding values of CTOD. CTOD values from the finite element analyses were compared to CTOD values predicted by PD6493, a strip-yield model and a line-spring model.

For a specified wall stress, wall strain, or applied moment, the severity of the crack sizes (in terms of applied CTOD) increases with crack depth -- $a/t=0.5$ is the most severe followed by $a/t=0.25$ then $a/t=0.125$. The numerical results clearly demonstrated the impact of depth for very long flaws.
The toughness levels of 0.005 in. or 0.010 in. CTOD for welds required in API-1104:Appendix A are sufficient to develop substantial plastic deformation before applied CTOD values reach these levels. In the two deepest cracks analyzed, the Net Ligament Yield (NLY) event occurs at approximately the 0.005 in. CTOD level. For the very shallow crack, \( a/t = 0.125 \), the CTOD attains a value of only 0.002 in. at NLY which occurs simultaneously with Gross Section Yielding. Because the NLY event occurs before the applied CTOD reaches the minimum required material toughness, other failure mechanisms besides ductile fracture, including plastic collapse and/or buckling, may govern the maximum loading.

CTOD values due to axial strain implied in the use of Fig. A5 in Appendix A were compared to the finite element estimates for CTOD. Finite element CTOD values were increased to account for residual strain effects. For the short, deep flaw (Case C), Fig. A5 significantly overestimates the applied CTOD (by factors \( > 3 \)) and unnecessarily limits the magnitude of allowable strain. For the long, shallow flaw (Case A), Fig. A5 overestimates the applied CTOD for axial strain levels less than \( \epsilon_y \). For larger axial strains, Fig. A5 underestimates the applied CTOD which increases rapidly due to the through-thickness shear bands. However, this may not lead to difficulties since most pipeline design codes limit the applied strain to \( 0.8 - 0.9 \epsilon_y \).

For cracks having \( a/t < 0.125 \), Appendix A appears to be especially conservative in view of the finite element results that show only Gross Section Yield for such situations. Very large wall strains (\( > 2.0 \epsilon_y \)) are needed to obtain CTOD values approaching 0.005 in.

The two tests, Nos. 18 and 19, in the series of 47 tests conducted by the WIC showed the apparently unconservative evaluation of long, shallow flaws using PD6493. Neither of these tested pipes qualify for assessment using API-1104:Appendix A due to the low CTOD values (0.004 in.) obtained for the girth weld metal. Perhaps more importantly, the weld metal flow stress undermatched the base metal flow stress by 6% which may have allowed additional deformation to concentrate at the crack.
CHAPTER 4

EXPERIMENTAL VERIFICATION

4.1. GENERAL

To obtain some experimental validation of the numerical studies described in the previous chapter, one full-scale pipe test was performed. The Welding Institute of Canada (WIC) was contracted by API to perform the test. A long, shallow flaw, very similar to configuration C of the parametric study, was placed in the base metal of an X-60 pipe having a 36 in. diameter and 0.9 in. wall thickness. The pipe was instrumented with gages to measure opening displacements of the crack faces, and with strain gages to measure the distribution of internal and external surface strains near the center of the crack. The pipe was loaded to failure in bending using the test procedures developed at WIC for the previous study [36] of cracks in pipeline girth welds.

A finite element analysis of the pipe was performed following the test. The exact dimensions of the crack were not known until the pipe was broken after the test. The finite element model and solution procedures followed closely those used for the parametric study.

This chapter presents detailed comparisons between the experimentally measured response of the tested pipe and the finite element predictions. The comparison focuses on the overall load-deflection response, the opening of the crack faces, and the axial strains on the pipe surface near the plane of the crack. Brief descriptions of the testing facility/procedure and the finite element model are also provided.

4.2. EXPERIMENTAL PROCEDURE

4.2.1. Testing Facility

The WIC facility is capable of testing pipes up to a diameter of 56 in. under bending. The testing frame uses a 40 ft. long section of pipe, which is loaded by a series of twenty, 50 ton hydraulic jacks placed on 2 ft. centers. The
hydraulic jacks are positioned to apply a distributed load through a series of saddles which are designed to prevent local buckling and to remove the need for internal pipe reinforcement. The end conditions of the pipe are designed to simulate simple supports (no moment). An elaborate restraint and shock absorbing system allows rapid failure of the defect region should that occur. The jack pressure is supplied by an electrically driven hydraulic pump and is controlled by a servo valve and servo controller. The applied loading creates a nearly constant moment region at center span where the flaw is located.

4.2.2. Pipe Geometry and Material Properties

A 17 ft. long section of an X-60 pipe with a 36 in. outside diameter and wall thickness of 0.9 in. was obtained by WIC for the testing program. In order to load this pipe in the testing facility, additional pipe sections were purchased and welded onto the test pipe on both ends to obtain the required 40 ft. specimen length. This carrier pipe was fabricated from X-70 material, but was thinner than the test pipe. Nominal thickness for the carrier pipe was 0.63 in. The carrier pipe was determined to be acceptable for the testing program given the lower moments away from the center of the pipe and the higher yield stress [36].

Material was taken from the test pipe for tensile tests and CTOD tests. The tensile coupons were standard 0.505 in. longitudinal specimens tested in accordance with ASTM E-8 [3]. The CTOD tests were performed according to BS-5762 [7] using specimens with a square cross-section of dimension equal to the pipe thickness. Notch depths with $a/W$ ratios of 0.2 and 0.5 were tested. The results of these tests are summarized in Table 6. The chemical composition and mill test results for the test pipe and carrier pipe are given in Table 7. The stress-strain curve obtained from the tensile tests included data up to a strain of approximately 1% and is shown in Fig. 27. Examination of the test pipe showed that the actual wall thickness to be 0.9 in. rather than the nominal value of 0.875 in. reported in Table 6.

4.2.3. Flaw Geometry

An external circumferential crack was placed in the pipe by pre-sawing a flaw of 0.06 in. depth using a series of interlinking slits. Fatigue loading of the
Table 6

Results of Tensile and CTOD Tests of Test Pipe

<table>
<thead>
<tr>
<th></th>
<th>Yield Strength (ksi)</th>
<th>Ultimate Strength (ksi)</th>
<th>Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.2% Offset</td>
<td>0.5% Offset</td>
<td></td>
</tr>
<tr>
<td>Test 1</td>
<td>56.1</td>
<td>63.8</td>
<td>89.9</td>
</tr>
<tr>
<td>Test 2</td>
<td>56.2</td>
<td>63.9</td>
<td>90.0</td>
</tr>
<tr>
<td>Average</td>
<td>56.15</td>
<td>63.85</td>
<td>89.9</td>
</tr>
</tbody>
</table>

|                 | CTOD (in.) |                  |              |              |
|-----------------|------------|------------------|--------------|
|                 | s/W = 0.5  | s/W = 0.2        |              |              |
| Test 1          | 0.0196     | 0.0376           |              |              |
| Test 2          | 0.0231     | 0.0338           |              |              |
| Test 3          | 0.0220     | 0.0379           |              |              |
| Average         | 0.0219     | 0.0364           |              |              |

crack progressively around the circumference at five locations produced the desired size of sharp crack. The final dimensions were obtained by breaking the pipe after the test and directly measuring the flaw. The measured flaw depth at the center was 0.118 in.; the flaw length as measured on the outside surface was 13.75 in.. These dimensions comply with ASTM E-399 [2] for fracture toughness testing which requires that at least 1.24 mm (0.050 in.) of the final crack be produced through fatigue growth. The resulting flaw size parameters are \( a/t = 0.13 \) and \( L/D = 0.38 \).

4.2.4. Instrumentation

Instrumentation for the test consisted primarily of strain gages located on the inside and outside tensile axes (see Fig. 28), clip gages, potential drop and pressure transducers. A total of 90 strain gages were installed. The gages were concentrated to measure the longitudinal strain of the middle 2 inches over the outside tensile axes and the middle 4 inches on the inside tensile axes of the pipe. Detailed schematics of the strain gage locations are documented in reference [36]. The outside strain gage locations near the flaw are shown in Fig. 28.
Table 7

Pipe Chemical Composition and Mill Test Reports

<table>
<thead>
<tr>
<th>Chemical</th>
<th>Test Pipe (X-60)</th>
<th>Carrier Pipe (X-70)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.20</td>
<td>0.07</td>
</tr>
<tr>
<td>Mn</td>
<td>1.16</td>
<td>0.44</td>
</tr>
<tr>
<td>Si</td>
<td>0.44</td>
<td>0.29</td>
</tr>
<tr>
<td>S</td>
<td>0.007</td>
<td>0.005</td>
</tr>
<tr>
<td>P</td>
<td>0.008</td>
<td>0.005</td>
</tr>
<tr>
<td>Nb</td>
<td>0.025</td>
<td>0.05</td>
</tr>
<tr>
<td>V</td>
<td>0.086</td>
<td>0.05</td>
</tr>
<tr>
<td>Ni</td>
<td>-</td>
<td>0.01</td>
</tr>
<tr>
<td>Cr</td>
<td>-</td>
<td>0.15</td>
</tr>
<tr>
<td>Mo</td>
<td>-</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Mill Test

<table>
<thead>
<tr>
<th>Nominal Dimensions</th>
<th>36 x 0.875 in.</th>
<th>36 x 0.63 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield Strength</td>
<td>63.1 ksi</td>
<td>75.8 ksi</td>
</tr>
<tr>
<td>Ultimate Strength</td>
<td>89.0 ksi</td>
<td>87.4 ksi</td>
</tr>
</tbody>
</table>

Figure 29 shows a simplified diagram of the clip gage setup used to monitor the crack face opening. The knife edges are bonded to the pipe surface by an epoxy substance applied over the entire contact area. For experimental verification, the displacement at the location of the knife edge center in the finite element model was used to compare against the experimental clip gage displacements (see Fig. 29).

4.2.5. Test Procedure

The pipe was loaded by increments of applied jack pressure. Since considerable plastic deformation was desired for comparison with the finite element computations, the test was carried out at room temperature. Potential drop measurements across the remaining ligament indicated no crack growth during the test.
The relationship between the jack pressure and the applied moment at center span for the test pipe is given by

\[ M \text{ (kip-ft)} = 13.23 \text{ } P \text{ (psi)} \]  

as derived in Appendix A of reference [36]. Figure 30 summarizes the analysis used to relate the jack pressures to the applied moment levels for the test pipe.

Figure 31 shows a typical plot of remote strain against jack pressure as reported in reference [36]. The strain values correspond to strain gage #3 located on the outside pipe surface as shown in Fig. 28. Evaluation of the strain gage plots in Appendix B of reference [36] show that no strain is recorded for jack pressure readings of less than 600 psi. This observation is consistent for all the strain gage plots and is attributed to seating adjustments of the hydraulic loading system. All finite element comparisons are based on experimental data that is shifted to correct for the 600 psi of jack pressure of seating adjustment.

As described in reference [36], the pipe suffered a buckling failure occurred in the region of the weld joining the test pipe to the carrier pipe. This corresponds to the location of maximum moment in the carrier pipe. The jack pressure at failure is reported to be 6700 psi which corresponds to a 6100 psi pressure once the seating error is taken into account. Using the linear relations between jack pressure and applied moment, this corresponds to a maximum moment in the test pipe of 1.69 times the yield moment.

4.3. FINITE ELEMENT MODEL

The finite element model for the test pipe was generated following the same procedure outlined in Chapter 2. A modified and more detailed mesh was introduced along the crack front. The elliptical mapping of a semi-circular crack onto a flat plate was not performed. Instead, the semi-elliptical crack was generated directly in the flat plate model. To accurately track development of the yield zone through the thickness, additional mesh refinement was included near the crack in the axial direction for a distance equal to the wall thickness.

The resulting finite element model contained a total of 468 elements and 2635 nodes. Two types of isoparametric elements were used: 20-node brick
elements with 3 by 3 by 3 Gauss integration and 15 node wedge elements with
7 area by 3 Gauss integration. Wedge elements were used to accomplish the
mesh transition near the end of the crack.

A linear analysis of the test-pipe model was performed using the pro-
cedures described in Section 3.3.1. The calculated stress intensity factors
showed agreement with the edge crack plate solutions comparable to that for
the parametric study models.

The nonlinear material behavior was modeled using incremental plasticity
described in Chapter 2. The stress-strain curve obtained from the tensile tests
recorded data to a strain of approximately 1%. For the finite element analysis
the remainder of the curve was input as a linear segment between the final
point on the test curve and the ultimate load. Figure 27 shows the segmental
stress strain curve employed in the finite element analysis.

The solution parameters were modified to update the stiffness matrix
before iterations 3, 6 and 9, with a limit of 10 equilibrium iterations. A con-
vergence tolerance of 0.1% for the Euclidean norm of the residual nodal loads
was used. The loading was applied in 13 variable increments; the solution for
each load step converged in the 3rd Newton iteration.

The POLO-FINITE [14,15] software was used for the analysis. Typical
CPU time for a load step requiring 3 Newton iterations to converge was 136
minutes on a Harris 1000 computer.

4.4. COMPARISON OF RESULTS

4.4.1. Load-Deflection Response

Figure 32 shows the remote strain versus pressure for strain gage #3 (Fig.
28.) corrected for the 600 psi seating error. The calculated load deflection
response for an uncracked pipe, both linear and nonlinear, and the finite ele-
ment results are also shown on the same figure. The response of the nonlinear
uncracked pipe was calculated using a nonlinear filament model as described in
Chapter 3. The linear response was obtained by standard beam theory calcula-
tions using a modulus of elasticity of 29000 ksi.
As shown on Fig. 32, the finite element results closely match the uncracked pipe solution. This is due to the negligible effect of the flaw on the overall strength of the pipe.

The finite element solution matches the experimental data well for jack pressures less than \( \approx 4000 \) psi. For additional increments of jack pressure, the finite element model predicts larger increases in remote strain than are measured experimentally.

Figure 33 shows the same data normalized with the axes positioned such that load is plotted on the vertical axis and deflection is plotted on the horizontal axes. The applied load is normalized by the moment at which yielding first occurs. The strain is normalized by the yield strain. For normalization purposes, the 0.2% offset yield stress of 56.1 ksi was used which corresponds to a yield strain of 0.193%. Young's modulus is 29000 ksi. The moment at first yield of the extreme fibers for an uncracked pipe is 47680 kip-in. As shown on this figure, the finite element results match the experimental data well to about 1.1 \( M_y \).

4.4.2. CMOD Response

Figure 34 shows the experimentally measured displacements and the finite element predictions plotted against the remote strains. The displacements measured at the clip gage are matched well by the finite element values. It is again noted that the experimentally measured clip gage displacements are not the crack mouth opening displacements (see Fig. 29).

The linear solution for CMOD was obtained from [35] for a SEN panel, with the crack length taken as the actual depth of the surface crack, \( a = 0.118 \text{ in.} \). The finite element predictions of CMOD closely match the linear solution values at lower strain levels. As can be seen, the measured clip gage displacements are given by \( \approx 1.7 \times \text{CMOD} \).

Figure 35 shows the same data plotted against the normalized moment. The finite element results match the data well up to a loading of about 1.1 \( M_y \).

For comparison purposes, the moments required to obtain the experimental remote strains were calculated using the nonlinear filament model for the
uncracked pipe. This was done in order to eliminate the error in measuring the experimental moment at the higher loadings. Figure 36 shows the data of Fig. 34 with the calculated moments corresponding to the remote strains. The finite element results match the experimental data very well.

Figures 37 and 38 show the finite element CMOD and CTOD values versus the normalized strain and moment, respectively. CTOD values were obtained using the $90^\circ$ intercept method documented in Chapter 2.

The largest clip gage displacement of 0.009 in. measured in the test corresponds to an actual CMOD of 0.005 in., and a CTOD of 0.0028 in. according to the finite element results.

4.4.3. Surface Strains

Strain distributions measured on the internal and external surfaces of the test pipe and the finite element predictions are shown on Figs. 39 and 40. For comparison purposes, the experimental strain distribution for the load of 1.2 $M_y$ was obtained from the strain gage plots by matching the experimental remote strains to the finite element results. This was done in order to eliminate the error in measuring the experimental moment at the higher loadings. Both the inside and outside strain distributions are matched reasonably well by the finite element results at these distances from the crack plane.

The strain distributions on the inside surface for loads of 0.5 $M_y$ and 0.9 $M_y$ are matched reasonably well by the finite element predictions. At a load of 1.2 $M_y$, a strain spike becomes apparent in the finite element solution at a distance of 0.45 inches from the crack plane. At this load, the experimental results show no appreciable spike, with a gradually decreasing strain from the remote strain value to the value at the crack center plane. The strain at the crack plane is matched by the finite element solution at all three load levels.

For the outside strains, the general trend toward zero at the crack mouth is evident for both the finite element solution and the experimental results.
maximum length shallow flaw \((a/t=0.25, \ L/D=0.4)\) permitted in Appendix A. The third crack size represents a very shallow flaw \((a/t=0.125)\) of maximum length \((L/D=0.4)\). Analysis of this third configuration provided a basis to assess the relative importance of crack depth for initially long flaws.

The 3-D finite element analyses were performed on very detailed models constructed of isoparametric solid elements combined with an incremental plasticity model for nonlinear material response. Singularity elements, formed by degenerating 20-node parabolic elements, were placed along the crack front. Finite element values for CTOD were obtained with the 90° intercept technique applied to the initially coincident nodes of the singularity elements along the crack front. Material properties typical of an X-70 line pipe were adopted for each analysis. The finite element models were loaded in bending by a force-couple applied at the remote end. Even though the crack was located in the base metal for the finite element analyses, the results are applicable to flaws in girth welds unless there is significant undermatching of the base metal yield strength by the weld metal. For overmatching situations the finite element values for CTOD should constitute an upper bound, especially for the two shallow cracks considered.

The finite element analyses established relationships between increasing values of CTOD and average wall stress, average wall strain and bending moment for the three crack configurations. The extent of plastic deformation in the pipes was related to the corresponding values of CTOD. Specifically, the levels of applied CTOD at the development of complete yielding across the remaining wall thickness (Net Ligament Yield, NLY) were determined in view of the 0.005 in. and 0.010 in. CTOD pre-qualified toughness levels required for application of Appendix A.

In addition to the finite element values, estimates for the applied CTOD were computed using three other elastic-plastic models: (1) PD6493 treated as an analysis tool to predict CTOD, (2) a modified strip-yield model, and (3) a simplified line-spring model. Comparisons were made for CTOD values given by these models and the finite element models as a function of average wall stress.
The crack driving force computation in API-1104:Appendix A, represented by Fig. A5, was used to obtain CTOD values for comparison with finite element results. Finite element values were adjusted to include a contribution from residual strains that would be present in a girth weld (Fig. A5 includes residual strain effects). Evaluation of the flaw assessment procedures in Appendix A also included a qualitative discussion of the conservatism introduced through use of a deeply notched, three-point bend bar to measure critical CTOD values and potential benefits of yield strength overmatching of the base metal by the girth weld metal.

To provide experimental validation of the finite element analyses, one full-scale test of a pipe containing a long, shallow surface flaw in the base metal was performed by the Welding Institute of Canada. The tested pipe had a 36 in. diameter and 0.9 in. wall thickness and thus closely matched the pipe geometry used in the parametric study. The flaw dimensions in the tested pipe were \( a/t = 0.13 \) and \( L/D = 0.38 \). The pipe was loaded slowly in bending at room temperature to allow development of extensive plasticity prior to failure. The pipe failed by buckling without detectable crack growth. A 3-D finite element model for the tested pipe was constructed and analyzed. Detailed comparisons were presented for the overall load-deflection response, opening displacements between the crack faces, and for axial strains on the inside and outside surfaces near the crack plane.

5.2. CONCLUSIONS

Results of the finite element and experimental studies described in this report support the following conclusions:

1. The maximum length deep flaw \((a/t = 0.5, \ L/t = 4)\) and the maximum length shallow flaw \((a/t = 0.25, \ L/D = 0.4)\) permitted in API-1104:Appendix A exhibit comparable CTOD values at the center of the cracks when subjected to the same wall stress, wall strain, or bending moment. The deep flaw has slightly larger CTOD values indicating it is the more severe of these two crack configurations.
2. The very shallow flaw \((a/t=0.125)\) exhibits significantly smaller CTOD values than computed for the two deeper crack configurations, especially after the applied strain exceeds the yield strain. For this crack, Gross Section Yielding, rather than Net Ligament Yielding, characterizes the nonlinear response. Gross Section Yielding is the general pattern of yielding exhibited by an uncracked pipe in bending. An average wall strain exceeding twice the yield strain is required to produce a CTOD approaching 0.005 in. Consequently, very shallow cracks \((a/t < 0.125)\) do not appear to be of major concern for failure by fracture.

3. For the crack sizes and material properties used in this study, the Net Ligament Yield condition develops before the applied CTOD values reach 0.005 in. Appendix A requires a minimum toughness of 0.005 in. and thus insures a significant level of ductile behavior.

4. Use of Fig. A5 in Appendix A for assessment of the long, shallow crack \((a/t=0.25, \ L/D=0.4)\) is conservative for applied axial strains up to the material yield strain. For strains exceeding yield, the CTOD values implied in the use of Fig. A5 are less than the finite element results. Under these conditions the overall safety factor for the flaw assessment procedure is reduced. However, most pipeline design codes limit the applied axial strain to a value less than the material yield strain.

5. For the deep crack \((a/t=0.5)\), Fig. A5 appears to unnecessarily limit the magnitude of strain due to external loading that can be safely tolerated. The variances in strains allowed by Fig. A5 reflect an attempt to match a complex range of nonlinear responses for different crack sizes with a relatively simple procedure.

6. The full-scale pipe test for a shallow flaw located in the base metal verified the 3-D, nonlinear finite element analyses employed in the parametric study. Comparisons of computed and measured crack face opening displacements and axial strains near the crack plane showed good agreement for nominal wall strains exceeding twice the yield strain.
7. Current evidence suggests that the use of three-point bend specimens with $a/W = 0.5$ to measure material toughness leads to additional conservatism in applications of Appendix A. The deeply cracked, three-point bend specimen imposes maximum constraint at the crack tip and underestimates critical CTOD values for shallow flaws by factors of 2 or more. When tension rather than bending is predominant on the remaining ligament (as for shallow flaws), critical CTOD values may be even greater.

8. When the flaw is located within the girth weld, overmatching of base metal yield strength by the weld metal yield strength appears to be beneficial. Overmatching reduces the applied CTOD values in shallow cracks but has little effect for larger cracks. The precise definition of a shallow crack in the context of overmatching conditions is not yet known.

9. The finite element and experimental results confirm the fundamental approach for flaw assessment in Appendix A for both short, deep flaws and long, shallow flaws. The application of Fig. A5 with critical CTOD values obtained from deeply notched bend bars is conservative. The degree of conservatism varies with different crack sizes as demonstrated in this study.

10. The two tests (Nos. 18 and 19) of long, shallow flaws performed by the WIC in the series of 47 tests do not qualify for assessment using API-1104:Appendix A due to low CTOD values (0.004 in.) for the girth weld metal. Moreover, the girth weld metal undermatched the base metal by 6%, which is also not permitted in Appendix A. The undermatching caused strains in the girth weld to be larger than strains measured in the pipe wall by some unknown amount. Consequently, the reported failure strains are too low by some amount which gives rise to the apparently low safety factors.
LIST OF REFERENCES


26. PATRAN-II Modeling Software. PDA Engineering, 1560 Brookhollow Dr., Santa Ana, Calif. 92705.


Fig. 1 General Pipe Arrangement and Loading
Fig. 2  Qualitative Flaw Assessment Diagram with API-1104:Appendix A Cutoffs on Flaw Sizes
Fig. 3  Section of Pipe for 3-D Finite Element Analysis
\[
t = 0.8 \text{ in.}
\]
\[
R = 17.2 \text{ in.}
\]
\[
D = 36.0 \text{ in.}
\]

<table>
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<th>Model Id</th>
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<th>2c (L)</th>
<th>L/D</th>
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</tr>
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<td>0.09</td>
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</tr>
</tbody>
</table>

Fig. 4 Crack Description Parameters and Parameter Values for the Crack Configurations Analyzed with Finite Element Models
Fig. 5a 3-D Finite Element Model Showing Remote and Near Crack Regions

36 in. = Dia.
Fig. 5b  Element Mesh at Each Point on Crack Front Showing Singularity Elements
X70 Line Pipe

E: 29,000 ksi
Prop. Limit: 85.5 ksi
Flow Stress: 92.3 ksi

Fig. 6 Engineering σ-ε Curve and Properties for X70 Line Pipe Steel Used in Finite Element Analyses
Fig. 7  90-Degree Intercept Definition of CTOD Used in Finite Element Analyses
3-D FINITE ELEMENT MODEL: BENDING
Material X70

Fig. 8 Load–Deflection Response for Crack Configuration A
3-D FINITE ELEMENT MODEL : BENDING
Material X70

( ) - CTOD

\[
\begin{align*}
(0.0042 \text{ in.}) \\
(0.0025 \text{ in.}) \\
(0.0017 \text{ in.}) \\
(0.0012 \text{ in.}) \\
(0.00083 \text{ in.}) \\
(0.00040 \text{ in.}) \\
(0.00019 \text{ in.}) \\
(0.00006 \text{ in.})
\end{align*}
\]

\[\text{t} = 0.8 \text{ in.; } R = 17.2 \text{ in.} \]
\[\text{a} = 0.1 \text{ in.; } 2c = 14.4 \text{ in.}\]

Fig. 9 Load–Deflection Response for Crack Configuration B
3-D FINITE ELEMENT MODEL: BENDING
Material X70

( ) - CTOD

\[(0.0421 \text{ in.})\]
\[(0.0274 \text{ in.})\]
\[(0.0102 \text{ in.})\]
\[(0.0046 \text{ in.})\]
\[(0.0033 \text{ in.})\]
\[(0.0021 \text{ in.})\]
\[(0.0008 \text{ in.})\]
\[(0.0002 \text{ in.})\]

\[t = 0.8 \text{ in.}; R = 17.2 \text{ in.}\]
\[a = 0.4 \text{ in.}; 2c = 3.2 \text{ in.}\]

Fig. 10 Load-Deflection Response for Crack Configuration C
Fig. 11 Illustration of Plastic Zone Development: (a) Plasticity Contained by Elastic Region (b) Plasticity Unconfined at Net Ligament Yield (NLY), (c) Gross Section Plasticity (GSY)
3-D FINITE ELEMENT MODEL : BENDING
Material X70

Axial Strain Distribution on Inside Surface of Pipe

\[ t = 0.8 \text{ in.}; \quad R = 17.2 \text{ in.} \]
\[ a = 0.2 \text{ in.}; \quad 2c = 14.4 \text{ in.} \]
\[ \theta = 0^\circ \]

Distance from Crack Plane (in.)

Strain \( \times 10^3 \)

Distance from Crack Plane (mm)

\[ M/M_f = 0.936, \quad \text{CTOD} = 0.0049 \text{ in.} \]
\[ M/M_f = 1.041, \quad \text{CTOD} = 0.0103 \text{ in.} \]
\[ M/M_f = 1.270, \quad \text{CTOD} = 0.0378 \text{ in.} \]

Fig. 12a Axial Strain Distribution on Inside Wall Surface at Center of Crack for Configuration A
Fig. 12b Axial Strain Distribution on Inside Wall Surface at Center of Crack for Configuration B
Fig. 13 CTOD Variation with Average Wall Stress for Each Crack Configuration
Fig. 14 CTOD Variation with Average Wall Strain for Each Crack Configuration
3-D FINITE ELEMENT MODEL: BENDING
Material X70

![Diagram]

- $\alpha/t = 0.25, 2c = 14.4$ in.
- $\alpha/t = 0.5, 2c = 3.2$ in.
- $\alpha/t = 0.125, 2c = 14.4$ in.

Fig. 15 CTOD Variation with Bending Moment for Each Crack Configuration
3-D FINITE ELEMENT MODEL: BENDING
Material X70

Fig. 16  CTOD Variation Along Crack Front for Crack Configuration A
3-D FINITE ELEMENT MODEL: BENDING
Material X70

Fig. 17  CTOD Variation Along Crack Front for Crack Configuration B
3-D FINITE ELEMENT MODEL: BENDING
Material X70

\[ t = 0.8 \text{ in.}; R = 17.2 \text{ in.} \]
\[ \alpha = 0.4 \text{ in.}; 2c = 3.2 \text{ in.} \]

Fig. 18 CTOD Variation Along Crack Front for Crack Configuration C
3-D FINITE ELEMENT MODEL : BENDING
Material X70

Fig. 19 Relationship Between CTOD and CMOD at Center of Crack for
Crack Configuration A
3-D FINITE ELEMENT MODEL: BENDING
Material X70

Fig. 20 Relationship Between CTOD and CMOD at Center of Crack for Crack Configuration B
3-D FINITE ELEMENT MODEL: BENDING
Material X70

Fig. 21 Relationship Between CTOD and CMOD at Center of Crack for Crack Configuration C
3-D FINITE ELEMENT MODEL: BENDING
Material X70

\[
t = 0.8 \text{ in.; } R = 17.2 \text{ in.}
\]
\[
a = 0.2 \text{ in.; } 2c = 14.4 \text{ in.}
\]

\[
\text{Avg. Wall Stress/Yield Stress}
\]

Fig. 22 Comparison of Finite Element, PD6493, Strip-Yield, and Line-Spring Estimates of CTOD for Crack Configuration A
3-D FINITE ELEMENT MODEL: BENDING

Material X70

\[ t = 0.8 \text{ in.}; \quad R = 17.2 \text{ in.} \]
\[ a = 0.1 \text{ in.}; \quad 2c = 3.2 \text{ in.} \]

Fig. 23 Comparison of Finite Element, PD6493, Strip-Yield, and Line-Spring Estimates of CTOD for Crack Configuration B
3-D FINITE ELEMENT MODEL: BENDING

Material X70

t = 0.8 in.; R = 17.2 in.
a = 0.4 in.; 2c = 3.2 in.

○ FE Model
□ Line-Spring Model
△ PD 6493
◊ Strip Yield

Fig. 24 Comparison of Finite Element, PD6493, Strip-Yield, and Line-Spring Estimates of CTOD for Crack Configuration C
Fig. 25 Ratio of CTOD Values Predicted by PD6493 to Finite Element Values for Crack Configurations A, B, and C
Given a

Interpolate to
Find CTOD

CTOD = 0.010 IN.

CTOD = 0.005 IN.

MAXIMUM APPLIED AXIAL STRAIN – $\varepsilon_a$

Fig. 26 Figure A5 Reproduced from API-1104:Appendix A
Fig. 27 Stress–Strain Curve for X60 Test Pipe
Fig. 28 Location of Strain Gages on Exterior Surface of Test Pipe
Fig. 29 Geometry of Clip-Gage to Measure Crack Opening Displacements
A = Jack Area = 11.025 sq. in.
p = Jack Pressure (psi)
P = Jack Load (lbs)
P = p A = 11.025 p
R = Reaction = 10 P
Max. Moment = 13.23 p (kip-ft)

Fig. 30  Jack Pressure vs. Applied Moment Relation for Test Pipe
Fig. 31 Measured Remote Strain (Gage #3) vs. Jack Pressure
Fig. 32  Comparison of Experimental, Finite Element, and Analytical Remote Strain as a Function of Jack Pressure
Fig. 33  Comparison of Experimental, Finite Element, and Analytical Applied Moment as a Function of Remote Strain
Fig. 34 Comparison of Experimental and Finite Element Crack Displacements as a Function of Remote Strain
Fig. 35 Comparison of Experimental and Finite Element Crack Displacements as a Function of Applied Moment
Fig. 36 Comparison of Experimental and Finite Element Crack Displacements as a Function of Applied Moment with Back Calculated Experimental Moment
Fig. 37 Relationship of CTOD and CMOD at Center of Crack vs. Remote Strains for Finite Element Results of Test Pipe
Fig. 38 Relationship of CTOD and CMOD at Center of Crack vs. Applied Moment for Finite Element Results of Test Pipe
Axial Strain Distribution
Pipe Inside Surface

Fig. 39 Comparison of Experimental and Finite Element Strains on Inside Surface of Pipe
Axial Strain Distribution
Pipe Outside Surface

Fig. 40 Comparison of Experimental and Finite Element Strains on Exterior Surface of Pipe