

# Recommended Provisions and Commentary on Development and Lap Splice Lengths for Deformed Reinforcing Bars in Tension

by David Darwin, LeRoy A. Lutz, and Jun Zuo

*Criteria recommended by ACI Committee 408 on development and lap splice length design for straight reinforcing bars in tension are presented in code format and compared with those in ACI 318-05, Building Code Recommendations for Structural Concrete. The recommended criteria produce designs with improved reliability compared to those in ACI 318. Development lengths are longer than those in ACI 318 for conditions of low cover or confinement, but shorter for bars with higher degrees of confinement, provided by added cover and transverse reinforcement and wider spacing between bars, and for normalweight concretes with strengths between 10,000 and 16,000 psi (70 and 110 MPa).*

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**Keywords:** bar ribs; bond; development length; high relative rib area; reinforcing bars; splice length.

## PREFACE

These recommended provisions and commentary are part of the work of ACI Committee 408, Bond and Development of Reinforcement, and represent proposed changes to Chapter 12 of ACI 318-05, "Building Code Requirements for Structural Concrete."<sup>1</sup> The provisions are based on the recommendations presented in ACI 408R-03<sup>2</sup> and are written to serve as a template for changes in Sections 12.2, 12.3, 12.15.1, and 12.15.2 of ACI 318, which cover development and lap splice design of straight bars in tension. In addition to provisions for conventional reinforcement, the proposed changes also include provisions for high relative rib area reinforcement.

The commentary is more extensive than is typical because of a desire to clarify the proposed provisions and provide appropriate comparisons with the provisions in ACI 318. Additional discussion is also presented following the Commentary for Sections 2.0 and 3.0 on development length and lap splices, respectively.

If adopted, the proposed provisions will provide designers with expressions that have improved levels of reliability and economy in comparison to the requirements in ACI 318-05. The improved reliability is based on better correlation between the design criteria in these provisions and bond test data compared to that obtained with the criteria in ACI 318. The proposed provisions also provide improved economy based on: (1) reductions in the number of situations in which Class B splices must be used; and (2) reduced development and lap splice lengths for bars with higher degrees of confinement, provided by transverse reinforcement, increased cover, and wider spacing between bars, and for normalweight concretes with strengths between 10,000 and 16,000 psi (70 and 110 MPa).

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## RECOMMENDED PROVISIONS and COMMENTARY

### 1.0—Notation

$A_{tr}$	=	total cross-sectional area of all transverse reinforcement that is within the spacing $s$ and crosses the potential plane of splitting through the reinforcement being developed or lap spliced, in. <sup>2</sup> (mm <sup>2</sup> )
$c_b$	=	$c_{min} + 0.5d_b$ , in. (mm)
$c_{bb}$	=	clear cover of reinforcement being developed or lap spliced, measured to tension face of member, in. (mm)
$c_{max}$	=	maximum value of $c_s$ or $c_{bb}$ , in. (mm)
$c_{min}$	=	minimum value of $c_s$ or $c_{bb}$ , in. (mm)
$c_s$	=	minimum value of $c_{si} + 0.25$ in. ( $c_{si} + 6$ mm) or $c_{so}$ , in. (mm). $c_{si}$ may be used in lieu of $c_{si} + 0.25$ in. ( $c_{si} + 6$ mm)
$c_{si}$	=	one-half of average clear spacing between bars or lap splices in a single layer, in. (mm)
$c_{so}$	=	clear cover of reinforcement being developed or lap spliced, measured to side face of member, in. (mm)
$d_b$	=	nominal bar diameter of developed or lap spliced bar, in. (mm)
$f'_c$	=	specified compressive strength of concrete, psi (MPa)
$f'_c^{1/4}$	=	fourth root of $f'_c$ , expressed in psi (MPa) units
$\sqrt{f'_c}$	=	square root of $f'_c$ , expressed in psi (MPa) units

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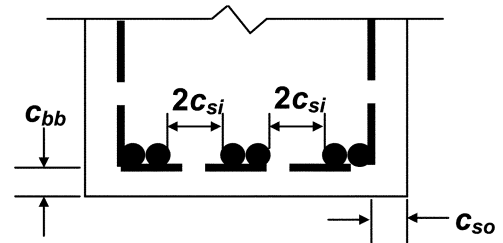


Fig. R1.1—Definition of  $c_{bb}$ ,  $c_{si}$ , and  $c_{so}$ .

- $f_{ct}$  = average splitting tensile strength of lightweight aggregate concrete, psi (MPa)
- $f_y$  = yield strength of reinforcement being developed or lap spliced, psi (MPa)
- $K_{tr}$  = transverse reinforcement index as defined in ACI 318-05, in. (mm)
- $K'_{tr}$  = transverse reinforcement index as defined by Eq. (2-3), in. (mm)
- $\ell_d$  = development length, in. (mm)
- $n$  = number of bars being developed or lap spliced along plane of splitting
- $R_r$  = relative rib area, ratio of projected rib area normal to bar axis to product of nominal bar perimeter and average center-to-center rib spacing
- $s$  = maximum center-to-center spacing of transverse reinforcement within  $\ell_d$ , in. (mm)
- $t_d$  = bar diameter factor for use in calculating  $K'_{tr}$ . Refer to 2.3
- $\psi_t$  = reinforcement location factor. Refer to 2.4
- $\psi_e$  = coating factor. Refer to 2.4
- $\lambda$  = lightweight aggregate concrete factor. Refer to 2.4
- $\omega$  = factor reflecting benefit of large cover/spacing perpendicular to controlling cover/spacing as defined by Eq. (2-2)

**R1.0—Notation**

- $A_r$  = projected rib area normal to reinforcing bar axis, in.<sup>2</sup> (mm<sup>2</sup>)
- $h_r$  = average height of deformations (measured according to S.2.3 of Appendix A), in. (mm)
- $s_r$  = average center-to-center rib spacing, in. (mm)

**2.0—Development of deformed reinforcing bars in tension**

**2.1**—Development length  $\ell_d$  for bars in tension shall be determined from either 2.2 or 2.3.

**2.1.1** When the load factors and strength reduction factors in Appendix C of ACI 318-05 are used, the development length  $\ell_d$  shall be reduced by multiplying by 0.85.

**2.1.2** In all cases,  $\ell_d$  shall not be less than the larger of  $16d_b$  or 12 in. (300 mm).

**2.1.3** The values of  $f'_c$ <sup>1/4</sup> used in these provisions shall not exceed 11.25 psi (3.25 MPa) for normalweight concrete or 10 psi (2.9 MPa) for lightweight concrete.

**2.1.4** The values of  $\sqrt{f'_c}$  used in these provisions shall not exceed 126 psi (10.5 MPa) for normalweight concrete or 100 psi (8.3 MPa) for lightweight concrete.

**2.2**—Unless determined based on 2.3, the development length in tension  $\ell_d$  shall be as follows:

Clear spacing of the bars being developed or spliced is not less than $d_b$ , and stirrups or ties throughout $\ell_d$ provide a value $K'_{tr}/d_b \geq 0.5$	$\left(\frac{f_y}{93f'_c{}^{1/4}} - 21\right) \psi_t \psi_e \lambda d_b$ (in.-lb)
or	$\left(\frac{f_y}{2.2f'_c{}^{1/4}} - 21\right) \psi_t \psi_e \lambda d_b$ (SI)
Clear spacing of the bars being developed or spliced is not less than $2d_b$ and clear cover is not less than $d_b$	
Other cases	$\left(\frac{f_y}{62f'_c{}^{1/4}} - 31\right) \psi_t \psi_e \lambda d_b$ (in.-lb)
	$\left(\frac{f_y}{1.5f'_c{}^{1/4}} - 31\right) \psi_t \psi_e \lambda d_b$ (SI)

**2.3**—The development length in tension  $\ell_d$  shall be

$$\ell_d = \frac{\left(\frac{f_y}{f'_c{}^{1/4}} - 2000\omega\right) \psi_t \psi_e \lambda}{62\left(\frac{c_b\omega + K'_{tr}}{d_b}\right)} d_b \text{ (in.-lb)} \quad (2-1)$$

$$\ell_d = \frac{\left(\frac{f_y}{f'_c{}^{1/4}} - 48\omega\right) \psi_t \psi_e \lambda}{1.5\left(\frac{c_b\omega + K'_{tr}}{d_b}\right)} d_b \text{ (SI)}$$

in which  $\left(\frac{c_b\omega + K'_{tr}}{d_b}\right)$  shall not exceed 4.

The factor  $\omega$  shall be taken as 1.0 or calculated as

$$\omega = 0.1 \frac{c_{max}}{c_{min}} + 0.9 \leq 1.25 \quad (2-2)$$

The transverse reinforcement index  $K'_{tr}$  shall be calculated as

$$K'_{tr} = \frac{t_d A_{tr} \sqrt{f'_c}}{2sn} \text{ (in.-lb)} \quad (2-3)$$

$$K'_{tr} = \frac{6t_d A_{tr} \sqrt{f'_c}}{sn} \text{ (SI)}$$

The bar diameter factor  $t_d$  shall be calculated as

$$t_d = 0.78d_b + 0.22 \text{ (in.-lb)} \quad (2-4)$$

$$t_d = 0.03d_b + 0.22 \text{ (SI)}$$

Alternatively, it shall be permitted to take  $K'_{tr} = 0$  as a design simplification, even if transverse reinforcement is present.

Using a value of  $c_{min}$  that is less than the actual value shall be permitted when calculating  $\ell_d$ . The same value of  $c_{min}$  shall be used when calculating  $\omega$  and  $c_b$ .

## R2.0—Development of deformed reinforcing bars in tension

The general development length equation (Eq. (2-1)) is based on work reported by ACI Committee 408.<sup>2</sup> Equation (2-1) is a reliability-based expression that produces a probability of failure in bond equal to approximately 1/5 of the probability of failure of a reinforced concrete beam in flexure. This is in contrast to the general development length equation in ACI 318-05 that produces a probability of failure in bond that is approximately equal to that of a flexural failure.<sup>2,3</sup> The higher proposed level of reliability is based on the brittle nature of a bond failure.

These provisions do not include the reinforcement size factor that is used in ACI 318-05 to reduce development and splice lengths of No. 6 (No. 19) and smaller bars because, as demonstrated in References 2 and 3, the application of such a factor provides very poor reliability.

Analysis of a database of development and splice specimens<sup>2,3</sup> supports the use of  $f'_c$ <sup>1/4</sup> to accurately represent the contribution of concrete strength to bond strength for bars not confined by transverse reinforcement and the use of  $f'_c$ <sup>3/4</sup> (shown as the product  $f'_c$ <sup>1/4</sup>  $\times$   $\sqrt{f'_c}$ ) to represent the contribution of concrete strength to the additional bond strength provided by transverse reinforcement. Further details are presented in Reference 2.

The limitations on the values for  $f'_c$ <sup>1/4</sup> and  $\sqrt{f'_c}$  are applicable for compressive strengths exceeding 16,000 psi (110 MPa) for normalweight concrete and 10,000 psi (69 MPa) for lightweight concrete. These limits reflect the upper range of test data available for epoxy-coated<sup>3,4</sup> as well as uncoated reinforcing bars at the time that these recommendations were developed.

The upper limit of 4 for the term  $(c_b\omega + K'_{tr})/d_b$  is based on the upper limit of confinement for which a splitting failure will occur. Higher values of confinement will not provide a commensurate increase in bond strength. This limit is higher than the upper limit of 2.5 that is applied to the term  $(c_b + K'_{tr})/d_b$  in ACI 318-05.

The 0.85 factor in Section 2.1.1 is introduced when the load factors and strength reduction factors in Appendix C of ACI 318-05 are used. This is done to maintain a level of reliability for bond that is consistent with the more conservative combination of load factors and tension strength reduction factor  $\phi$  employed in the Appendix. In other words, to maintain the same reliability, using the load factors in Chapter 9 of ACI 318-05, in conjunction with the continued use of  $\phi = 0.9$  for flexure and tension, requires the use of lap splice and

development lengths that are 18% longer than are required when using the higher load factors in Appendix C of ACI 318-05 with the same value of  $\phi$ .

The equations in Section 2.2 result from setting the parameter  $\omega = 1.0$  and setting  $(c_b\omega + K'_{tr})/d_b$  in Eq. (2-1) to 1.5 for the first equation and 1.0 for the second equation. Section 2.2 recognizes that many practical construction cases have spacing and cover values and confining reinforcement, such as stirrups or ties, that result in a value of  $(c_b\omega + K'_{tr})/d_b$  equal at least 1.5. Examples include a minimum clear cover  $d_b$  along with either a minimum clear spacing of  $2d_b$  or a combination of minimum clear spacing  $d_b$  with ties or stirrups providing  $K'_{tr}/d_b \geq 0.5$ .

As in ACI 318R-05, the user may easily construct simplified, useful expressions. For example, in all structures with normalweight concrete ( $\lambda = 1.0$ ), uncoated reinforcement ( $\psi_e = 1.0$ ), and bottom bars ( $\psi_t = 1.0$ ) with  $f'_c = 4000$  psi (28 MPa) and Grade 60 (Grade 420) reinforcement, the equations in Section 2.2 reduce to

$$\ell_d = \left( \frac{60,000}{93(4000)^{1/4}} - 21 \right) (1.0)(1.0)(1.0)d_b = 60d_b \text{ (in.-lb)}$$

$$\ell_d = \left( \frac{60,000}{62(4000)^{1/4}} - 31 \right) (1.0)(1.0)(1.0)d_b = 91d_b \text{ (in.-lb)}$$

$$\ell_d = \left( \frac{420}{2.2(28)^{1/4}} - 21 \right) (1.0)(1.0)(1.0)d_b = 62d_b \text{ (SI)}$$

$$\ell_d = \left( \frac{420}{1.5(28)^{1/4}} - 31 \right) (1.0)(1.0)(1.0)d_b = 91d_b \text{ (SI)}$$

For the load and strength reduction factors in Appendix C of ACI 318-05, these values are  $51d_b$  and  $77d_b$  ( $53d_b$  and  $77d_b$ ), respectively.

Many practical combinations of bar spacing, clear cover, and confining reinforcement can be used with Section 2.3 to produce significantly shorter development lengths than allowed by Section 2.2. The maximum amount of confinement allowed in Eq. (2-1), as represented by the term  $(c_b\omega + K'_{tr})/d_b$ , is equal to 4.0, compared with 2.5 for  $(c_b + K'_{tr})/d_b$  in ACI 318-05; this allows a further reduction in development length. For the concrete and reinforcement strengths used above,  $\omega = 1$ , and under conditions of high confinement with  $(c_b\omega + K'_{tr})/d_b = 4$ , Eq. (2-1) produces

$$\ell_d = \frac{\left( \frac{60,000}{(4000)^{1/4}} - 2000 \times 1 \right) (1.0)(1.0)(1.0)}{62(4)} d_b = 22d_b \text{ (in.-lb)}$$

$$\ell_d = \frac{\left( \frac{420}{(28)^{1/4}} - 48 \times 1 \right) (1.0)(1.0)(1.0)}{1.5(4)} d_b = 22d_b \text{ (SI)}$$

A value of  $c_{min}$  that is less than the actual value would be used to calculate  $\ell_d$  in cases where  $(c_b\omega + K'_{tr})/d_b$  is set to its maximum of 4. In these cases, increasing cover  $c_{bb}$  can result

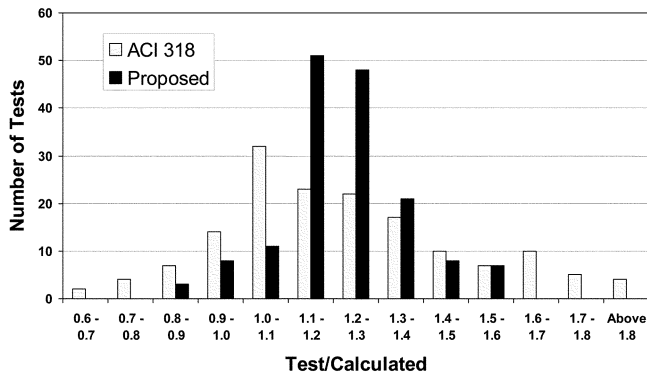


Fig. R2.1(a)—Test/calculated ratios for ACI 318-05 and proposed provisions for bars not confined by transverse reinforcement.

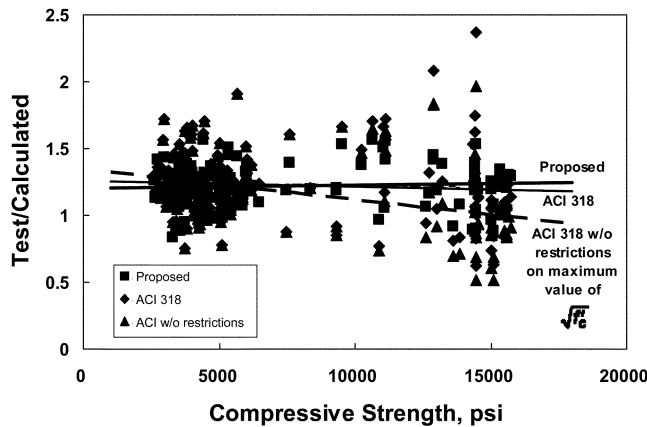


Fig. R2.1(b)—Test/calculated ratios versus compressive strength for bars not confined by transverse reinforcement (1 psi = 6.895 kPa).

in a requirement to increase, rather than decrease,  $l_d$ ; this occurs because the denominator in Eq. (2-1) is constant, while the numerator increases as  $\omega$  decreases because  $c_{max}$  (the spacing between bars) is constant and  $c_{min} = c_{bb}$  is increasing. If a lower than actual value of  $c_{min}$  is used, it must be used to calculate  $c_b$  as well as  $\omega$ .

**2.4**—The factors used in the expressions for development of bars in tension are as follows:

- $\psi_t$  = reinforcement location factor  
 Horizontal reinforcement so placed that more than 12 in. (300 mm) of fresh concrete is cast in the member below the development length or splice..... 1.3  
 Other reinforcement ..... 1.0

- $\psi_e$  = coating factor  
 Epoxy-coated bars ..... 1.5  
 Uncoated reinforcement..... 1.0

However, the product  $\psi_t\psi_e$  need not be taken greater than 1.7.

- $\lambda$  = lightweight aggregate concrete factor  
 When lightweight aggregate concrete is used..... 1.3  
 However, when  $f_{ct}$  is specified,  $\lambda$  shall be permitted to be taken as  $6.7\sqrt{f'_c}/f_{ct}$  ( $\sqrt{f'_c}/1.8f_{ct}$ ) but not less than ..... 1.0  
 When normalweight concrete is used ..... 1.0

**R2.4**—The factors in 2.4 are unchanged from those in ACI 318, with the exception that only a single value of  $\psi_e$  is

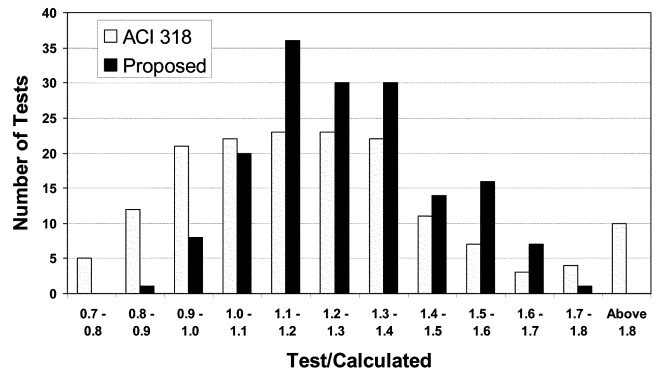


Fig. R2.1(c)—Test/calculated ratios for ACI 318-05 and proposed provisions for bars confined by transverse reinforcement.

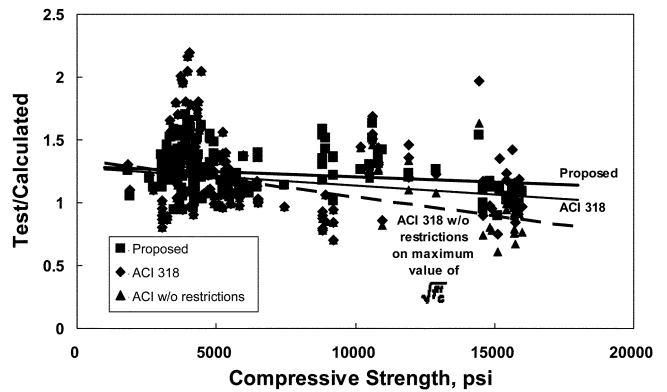


Fig. R2.1(d)—Test/calculated ratios versus compressive strength for bars confined by transverse reinforcement (1 psi = 6.895 kPa).

used for epoxy-coated reinforcement. The justification for the single value is presented in References 2 and 3.

**2.5 Excess reinforcement**—Reduction in development length shall be permitted where reinforcement in a flexural member is in excess of that required by analysis, except where anchorage or development for  $f_y$  is specifically required or the reinforcement is designed under provisions of 21.2.1.4 of ACI 318-05.....( $A_s$  required)/( $A_s$  provided)

**R2.5 Excess reinforcement**—The tests used to develop Eq. (2-1) demonstrate that a reduction in development length based on the ratio of  $A_s$  required to  $A_s$  provided is conservative when applied to the expressions for  $l_d$  in Sections 2.2 and 2.3. In such cases, application of the factor ( $A_s$  required)/( $A_s$  provided) results in a bond force that still exceeds the bar force corresponding to  $A_s$  required. This factor is not applied in cases where the full yield strength  $f_y$  is required or where the structure will be subjected to severe seismic loading.

**Additional discussion of Section 2.0:**

The improved accuracy provided by Eq. (2-1) compared with that provided by Eq. (12-1) in ACI 318-05 and the appropriateness of  $f'_c^{1/4}$  to represent the contribution of concrete strength to bond strength are demonstrated in Fig. R2.1(a) through (d). In the figures, data from ACI Committee 408 Database 2001-10<sup>2</sup> are used for comparison. Figure R2.1(a) compares the distribution of test-to-calculated ratios for bond forces based on ACI 318-05 with those for these provisions for bars not confined by transverse reinforcement. The figure demonstrates

**Table R2.1—Comparison of development lengths, expressed as  $l_d/d_b$ , for ACI 318-05 and proposed provisions for: (a) Grade 60 (414 MPa<sup>\*</sup>); and (b) Grade 75 (517 MPa<sup>†</sup>) bars**

(a) Grade 60 (414 MPa <sup>*</sup> ) reinforcement						(b) Grade 75 (517 MPa <sup>†</sup> ) reinforcement					
$(c_b + K_{tr})/d_b = 1.0$ for ACI 318-05, $(c_b + K'_{tr})/d_b = 1.0$ for proposed						$(c_b + K_{tr})/d_b = 1.0$ for ACI 318-05, $(c_b + K'_{tr})/d_b = 1.0$ for proposed					
$f'_c$ , psi	ACI 318-05 <sup>‡</sup>	Proposed Section 2.2	Proposed Section 2.3 $\omega = 1$	Proposed Section 2.3 $\omega = 1.25$	Proposed Section 2.2 and ACI 318-05 Appendix C	$f'_c$ , psi	ACI 318-05 <sup>‡</sup>	Proposed Section 2.2	Proposed Section 2.3 $\omega = 1$	Proposed Section 2.3 $\omega = 1.25$	Proposed Section 2.2 and ACI 318-05 Appendix C
3000	82.2	99.8	98.5	72.4	84.8	3000	102.7	132.5	131.2	98.5	112.6
4000	71.2	90.7	89.4	65.1	77.1	4000	88.9	121.1	119.9	89.4	102.9
5000	63.6	84.1	82.8	59.8	71.5	5000	79.5	112.9	111.6	82.8	96.0
6000	58.1	79.0	77.7	55.7	67.1	6000	72.6	106.4	105.2	77.7	90.4
7000	53.8	74.8	73.5	52.4	63.6	7000	67.2	101.2	100.0	73.5	86.0
8000	50.3	71.3	70.1	49.6	60.6	8000	62.9	96.9	95.6	70.1	82.4
10,000	45.0	65.8	64.5	45.2	55.9	10,000	56.3	90.0	88.7	64.5	76.5
12,000	45.0	61.5	60.2	41.7	52.2	12,000	56.3	84.6	83.3	60.2	71.9
15,000	45.0	56.4	55.2	37.7	48.0	15,000	56.3	78.3	77.0	55.2	66.6
$(c_b + K_{tr})/d_b = 1.5$ for ACI 318-05, $(c_b + K'_{tr})/d_b = 1.5$ for proposed						$(c_b + K_{tr})/d_b = 1.5$ for ACI 318-05, $(c_b + K'_{tr})/d_b = 1.5$ for proposed					
$f'_c$ , psi	ACI 318-05 <sup>‡</sup>	Proposed Section 2.2	Proposed Section 2.3 $\omega = 1$	Proposed Section 2.3 $\omega = 1.25$ $c_b/d_b = 1.0$	Proposed Section 2.2 and ACI 318-05 Appendix C	$f'_c$ , psi	ACI 318-05 <sup>‡</sup>	Proposed Section 2.2	Proposed Section 2.3 $\omega = 1$	Proposed Section 2.3 $\omega = 1.25$ $c_b/d_b = 1.0$	Proposed Section 2.2 and ACI 318-05 Appendix C
3000	54.8	66.2	65.7	51.7	56.3	3000	68.5	88.0	87.5	70.4	74.8
4000	47.4	60.1	59.6	46.5	51.1	4000	59.3	80.4	79.9	63.9	68.3
5000	42.4	55.7	55.2	42.7	47.3	5000	53.0	74.9	74.4	59.2	63.7
6000	38.7	52.3	51.8	39.8	44.5	6000	48.4	70.6	70.1	55.5	60.0
7000	35.9	49.5	49.0	37.4	42.1	7000	44.8	67.2	66.7	52.5	57.1
8000	33.5	47.2	46.7	35.4	40.1	8000	41.9	64.3	63.8	50.0	54.7
10,000	30.0	43.5	43.0	32.3	37.0	10,000	37.5	59.6	59.1	46.1	50.7
12,000	30.0	40.6	40.1	29.8	34.5	12,000	37.5	56.1	55.5	43.0	47.7
15,000	30.0	37.3	36.8	26.9	31.7	15,000	37.5	51.9	51.4	39.4	44.1
Maximum confinement: $(c_b + K_{tr})/d_b = 2.5$ for ACI 318-05, $(c_b\omega + K'_{tr})/d_b = 2.5$ and 4 for proposed						Maximum confinement: $(c_b + K_{tr})/d_b = 2.5$ for ACI 318-05, $(c_b\omega + K'_{tr})/d_b = 2.5$ and 4 for proposed					
$f'_c$ , psi	ACI 318-05 <sup>‡</sup>	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 2.5, \omega = 1$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 4, \omega = 1$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 4, \omega = 1.25$		$f'_c$ , psi	ACI 318-05 <sup>‡</sup> $(c_b + K_{tr})/d_b = 2.5$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 2.5, \omega = 1$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 4, \omega = 1$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 4, \omega = 1.25$	
3000	32.9	39.4	24.6	22.6		3000	41.1	52.5	32.8	30.8	
4000	28.5	35.8	22.4	20.3		4000	35.6	47.9	30.0	27.9	
5000	25.5	33.1	20.7	18.7		5000	31.8	44.6	27.9	25.9	
6000	23.2	31.1	19.4	17.4		6000	29.0	42.1	26.3	24.3	
7000	21.5	29.4	18.4	16.4		7000	26.9	40.0	25.0	23.0	
8000	20.1	28.0	17.5	16.0		8000	25.2	38.3	23.9	21.9	
10,000	18.0	25.8	16.1	16.0		10,000	22.5	35.5	22.2	20.2	
12,000	18.0	24.1	16.0	16.0		12,000	22.5	33.3	20.8	18.8	
15,000	18.0	22.1	16.0	16.0		15,000	22.5	30.8	19.3	17.2	

<sup>\*</sup>A soft conversion is used for this comparison because Grade 420 corresponds to 60,900 psi, requiring slightly longer development lengths than those listed for Grade 60 reinforcement.  
<sup>†</sup>A soft conversion is used for this comparison because Grade 520 corresponds to 75,400 psi, requiring slightly longer development lengths than those listed for Grade 75 reinforcement.  
<sup>‡</sup>Reinforcement size factor = 1.0.

that the proposed provisions produce a better match with the test results than do the provisions of ACI 318 (that is, more reliable results with less scatter). For the comparison shown in Fig. R2.1(a), 17% of the test/calculated ratios for ACI 318-05 fall below 1.0, compared with 7% for the proposed criteria. Figure R2.1(b) shows the relationship between the test and calculated values using ACI 318 without the maximum limit on  $\sqrt{f'_c}$  of 100 psi (8.33 MPa) (equivalent to a maximum usable compressive strength of 10,000 psi [70 MPa]), ACI 318 with the required maximum cutoff, and these provisions. The test-calculated ratio is shown to drop with increasing compressive strength for the basic relationship in

ACI 318, but to be essentially independent of compressive strength for ACI 318 once the maximum value on  $\sqrt{f'_c}$  is applied, thus emphasizing the importance of the upper limit on  $\sqrt{f'_c}$  when applying the provisions in ACI 318-05. The trend line based on these provisions is even less sensitive to  $\sqrt{f'_c}$ . Figure R2.1(c) and (d) show similar results for bars that are confined by transverse reinforcement. In Fig. R2.1(c), 23% of the test/calculated ratios for ACI 318-05 fall below 1.0, compared with 6% for the proposed criteria.

Table R2.1 compares development lengths for Grades 60 and 75 (414 and 517 MPa) bars based on ACI 318-05 (with a reinforcement size factor of 1.0) and these provisions

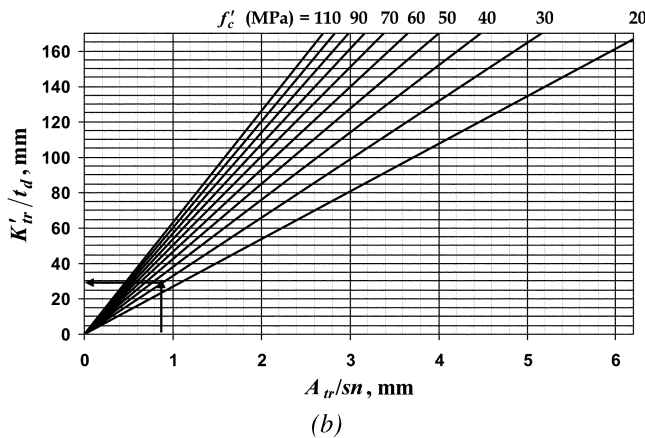
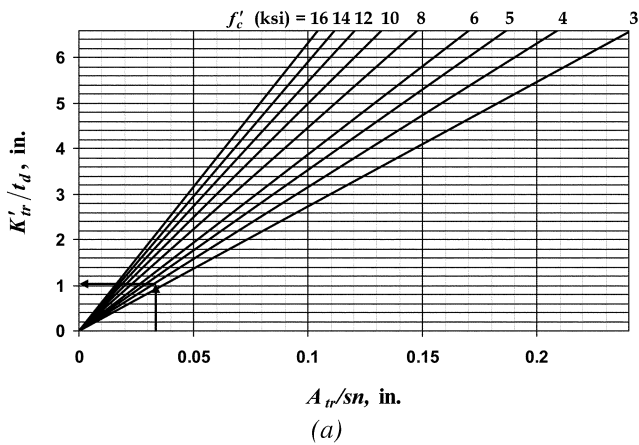


Fig. R2.2— $K'_t/t_d$  as function of  $A_{tr}/sn$  and  $f'_c$ : (a) in.-lb; and (b) SI.

(Sections 2.2 and 2.3) and covers cases ranging from minimum clear cover and bar spacing to cases with the maximum confinement that may be used for design in the respective provisions. As shown in the table, values for  $\ell_d$  given in Section 2.2 are greater than those obtained using Chapter 12 of ACI 318-05 at corresponding levels of spacing and confinement. Table R2.1 also demonstrates that development lengths  $\ell_d$  calculated using Section 2.3 decrease relative to those in ACI 318-05 as the degree of confinement and the concrete compressive strength increase and can be 30% or more below those required by ACI 318. One other set of comparisons can be made using Table R2.1—comparisons between the values of  $\ell_d$  based on ACI 318 and those based on Section 2.2 when the load factors of Appendix C of ACI 318-05 are used for design. In this case, the higher reliability in bond (as well as in flexure) attainable with the higher load factors in Appendix C allows  $\ell_d$  based on the proposed provisions to be reduced by 15%. The resulting values are only fractionally greater than the development lengths required by ACI 318-05, which were originally used with the load factors that appear in Appendix C through the 1999 edition of ACI 318.<sup>5</sup> A similar 15% decrease is available under Section 2.3.

Figure R2.2 and Table R2.2 are provided to assist in the calculation of  $K'_t$  in Section 2.2 or Section 2.3. The value  $K'_t$  can be calculated by taking the product of  $K'_t/t_d$ , which is based on  $A_{tr}/sn$  in Fig. R2.2, and the value of  $t_d$ , which is based on bar size, from Table R2.2. For example, for  $f'_c = 4000$  psi, if three closely spaced No. 9 bars ( $n = 3$ ) are confined by No. 4 bar stirrups with two legs spaced at  $s = 4$  in.,

Table R2.2— $t_d$  and  $t_d/d_b$  as functions of bar size

Bar size	Bar diameter, in. (mm)	$t_d$	$t_d/d_b$ , in. <sup>-1</sup>	$t_d/d_b$ , mm <sup>-1</sup>
No. 4 (No. 13)	0.500 (12.7)	0.61	1.22	0.048
No. 5 (No. 16)	0.625 (15.9)	0.71	1.13	0.044
No. 6 (No. 19)	0.750 (19.1)	0.81	1.07	0.042
No. 7 (No. 22)	0.875 (22.2)	0.90	1.03	0.041
No. 8 (No. 25)	1.000 (25.4)	1.00	1.00	0.039
No. 9 (No. 29)	1.128 (28.7)	1.10	0.98	0.038
No. 10 (No. 32)	1.270 (32.3)	1.21	0.95	0.037
No. 11 (No. 36)	1.410 (35.8)	1.32	0.94	0.037
No. 14 (No. 43)	1.693 (43.0)	1.54	0.91	0.036
No. 18 (No. 57)	2.257 (57.3)	1.98	0.88	0.035

then  $A_{tr}/sn = 0.20 \times 2/(4 \times 3) = 0.033$  in. From Fig. R2.2(a),  $K'_t/t_d = 1.05$  in. From Table R2.2,  $t_d = 1.10$  and  $t_d/d_b = 0.98$  in.<sup>-1</sup>. Thus,  $K'_t = (K'_t/t_d) \times t_d = 1.05 \times 1.10 = 1.16$  in., and  $K'_t/d_b = (K'_t/t_d) \times (t_d/d_b) = 1.05 \times 0.98 = 1.03$ . (For example, for  $f'_c = 30$  MPa, if three closely spaced No. 29 bars [ $n = 3$ ] are confined by No. 13 bar stirrups with two legs spaced at  $s = 100$  mm, then  $A_{tr}/sn = 129 \times 2/[100 \times 3] = 0.86$  mm. From Fig. R2.2(b),  $K'_t/t_d = 28$  mm. From Table R2.2,  $t_d = 1.10$  and  $t_d/d_b = 0.038$  mm<sup>-1</sup>. Thus,  $K'_t = [K'_t/t_d] \times t_d = 28 \times 1.10 = 30.8$  mm, and  $K'_t/d_b = [K'_t/t_d] \times [t_d/d_b] = 28 \times 0.038 = 1.06$ .) With  $K'_t$  or  $K'_t/d_b$  calculated, the development length  $\ell_d$  may be determined using the provisions of either Section 2.2 or Section 2.3.

### 3.0—Lap splices of deformed reinforcing bars in tension

**3.1**—Minimum length of lap for tension lap splices shall be as required for Class A, B, or C splices, but not less than  $16d_b$  or 12 in. (300 mm), where

- Class A splice.....  $1.0\ell_d$
- Class B splice.....  $1.0\ell_d$  with  $\omega = 1.0$
- Class C splice.....  $1.25\ell_d$  with  $\omega = 1.0$

where  $\ell_d$  is the tensile development length for the specified yield strength  $f_y$  in accordance with 2.2 or 2.3 without the modification factor in 2.5.

**R3.1**—Although tests demonstrate that splice strength where  $(A_s \text{ required})/(A_s \text{ provided}) < 1$  is conservatively represented by the modification factor in 2.5, this factor is not used to increase the level of reliability of lap splices.

**3.2**—Lap splices of deformed bars in tension shall be Class B splices, except that Class A splices shall be allowed when the criteria of 3.2.1, 3.2.2, or 3.2.3 are met or when Class C splices are required in accordance with 3.3.

**3.2.1**—When the spliced reinforcement is confined with transverse reinforcement at two or more locations with spacing  $s$  not greater than 12 in. (300 mm), providing that  $K'_t/d_b$  is at least 1.0.

**3.2.2**—When no more than one-half of the total reinforcement is spliced within the required lap length.

**3.2.3**—For horizontal reinforcing bars in walls that are not being used as in-plane flexural or tensile members.

**R3.2**—Tests<sup>2,3,6,7</sup> demonstrate that strength requirements are fulfilled by Eq. (2-1) for splices with a factor of 1.0, even

**Table R3.2—Comparison of Class B splice lengths, expressed as  $\ell_d/d_b$ , for ACI 318-05 and proposed provisions for: (a) Grade 60 (414 MPa<sup>\*</sup>); and (b) Grade 75 (517 MPa<sup>†</sup>) bars**

(a) Grade 60 (414 MPa <sup>*</sup> ) reinforcement				(b) Grade 75 (517 MPa <sup>†</sup> ) reinforcement			
$(c_b + K_{tr})/d_b = 1.0$ for ACI 318-05, $(c_b + K'_{tr})/d_b = 1.0$ for proposed				$(c_b + K_{tr})/d_b = 1.0$ for ACI 318-05, $(c_b + K'_{tr})/d_b = 1.0$ for proposed			
$f'_c$ , psi	ACI 318-05 <sup>‡</sup> $1.3\ell_d$	Proposed Section 2.2 $1.0\ell_d$ <sup>§</sup>	Proposed Section 2.3 $1.0\ell_d$ with $\omega = 1.0$	$f'_c$ , psi	ACI 318-05 <sup>‡</sup> $1.3\ell_d$	Proposed Section 2.2 $1.0\ell_d$ <sup>§</sup>	Proposed Section 2.3 $1.0\ell_d$ with $\omega = 1$
3000	106.8	99.8	98.5	3000	133.5	132.5	131.2
4000	92.5	90.7	89.4	4000	115.6	121.1	119.9
5000	82.7	84.1	82.8	5000	103.4	112.9	111.6
6000	75.5	79.0	77.7	6000	94.4	106.4	105.2
7000	69.9	74.8	73.5	7000	87.4	101.2	100.0
8000	65.4	71.3	70.1	8000	81.8	96.9	95.6
10,000	58.5	65.8	64.5	10,000	73.1	90.0	88.7
12,000	58.5	61.5	60.2	12,000	73.1	84.6	83.3
15,000	58.5	56.4	55.2	15,000	73.1	78.3	77.0
$(c_b + K_{tr})/d_b = 1.5$ for ACI 318-05, $(c_b + K'_{tr})/d_b = 1.5$ for proposed				$(c_b + K_{tr})/d_b = 1.5$ for ACI 318-05, $(c_b + K'_{tr})/d_b = 1.5$ for proposed			
$f'_c$ , psi	ACI 318-05 <sup>‡</sup> $1.3\ell_d$	Proposed Section 2.2 $1.0\ell_d$ <sup>§</sup>	Proposed Section 2.3 $1.0\ell_d$ with $\omega = 1$	$f'_c$ , psi	ACI 318-05 <sup>‡</sup> $1.3\ell_d$	Proposed Section 2.2 $1.0\ell_d$ <sup>§</sup>	Proposed Section 2.3 $1.0\ell_d$ with $\omega = 1$
3000	71.2	66.2	65.7	3000	89.0	88.0	87.5
4000	61.7	60.1	59.6	4000	77.1	80.4	79.9
5000	55.2	55.7	55.2	5000	68.9	74.9	74.4
6000	50.3	52.3	51.8	6000	62.9	70.6	70.1
7000	46.6	49.5	49.0	7000	58.3	67.2	66.7
8000	43.6	47.2	46.7	8000	54.5	64.3	63.8
10,000	39.0	43.5	43.0	10,000	48.8	59.6	59.1
12,000	39.0	40.6	40.1	12,000	48.8	56.1	55.5
15,000	39.0	37.3	36.8	15,000	48.8	51.9	51.4
Maximum confinement: $(c_b + K_{tr})/d_b = 2.5$ for ACI 318-05, $(c_b\omega + K'_{tr})/d_b = 2.5$ and 4 for proposed				Maximum confinement: $(c_b + K_{tr})/d_b = 2.5$ for ACI 318-05, $(c_b\omega + K'_{tr})/d_b = 2.5$ and 4 for proposed			
$f'_c$ , psi	ACI 318-05 <sup>‡</sup> $1.3\ell_d$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 2.5$ , $\omega = 1$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 4$ , $\omega = 1$	$f'_c$ , psi	ACI 318-05 <sup>‡</sup> $1.3\ell_d$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 2.5$ , $\omega = 1$	Proposed Section 2.3 $(c_b\omega + K'_{tr})/d_b = 4$ , $\omega = 1$
3000	42.7	39.4	24.6	3000	53.4	52.5	32.8
4000	37.0	35.8	22.4	4000	46.2	47.9	30.0
5000	33.1	33.1	20.7	5000	41.4	44.6	27.9
6000	30.2	31.1	19.4	6000	37.8	42.1	26.3
7000	28.0	29.4	18.4	7000	35.0	40.0	25.0
8000	26.2	28.0	17.5	8000	32.7	38.3	23.9
10,000	23.4	25.8	16.1	10,000	29.3	35.5	22.2
12,000	23.4	24.1	16.0	12,000	29.3	33.3	20.8
15,000	23.4	22.1	16.0	15,000	29.3	30.8	19.3

\* A soft conversion is used for this comparison because Grade 420 corresponds to 60,900 psi, requiring slightly longer development lengths than those listed for Grade 60 reinforcement.

† A soft conversion is used for this comparison because Grade 520 corresponds to 75,400 psi, requiring slightly longer development lengths than those listed for Grade 75 reinforcement.

‡ Reinforcement size factor = 1.0.

§  $\omega = 1.0$  for  $\ell_d$  in Section 2.2.

when all bars are spliced at the same location. Historically, however, a factor of 1.3 has been used for Class B splices to encourage less congested reinforcement details and to improve reliability.<sup>1</sup> A value of  $\omega = 1.0$  is used here for the same reasons when  $K'_{tr}/d_b$  is less than 1.0 and more than one-half of the reinforcement is spliced at one location.

The development length  $\ell_d$  properly reflects the required splice length for walls provided that the top bar factor of 1.3 is employed. There is no need to also require a Class B splice, as has been done in the past for walls where all bars are spliced at the same location.

**3.3**—Lap splices in tension tie members are permitted using a Class C splice, provided

- no more than one-half of the bars are spliced at any one location,
- $c_{min}$  is at least  $1.5d_b$ , and
- $A_{tr}/s_n$  is at least  $d_b/20$  with a ninety degree or greater bend in the transverse reinforcement confining each spliced bar.

**R3.3**—Although splicing of tension tie members with lap splices has not been permitted since the introduction of ACI 318-83,<sup>8</sup> it is, in fact, done in practice. With proper confinement of splices in tie members, it has been shown<sup>9</sup> that such members can behave very well, even under cyclic loading conditions.

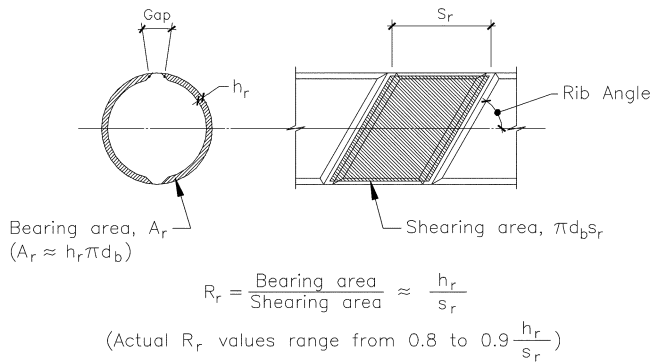


Fig. R4.1—Definition of  $R_r$ .<sup>10</sup>

#### Additional discussion of Section 3.0:

Table R3.2 compares lap splice lengths for Grade 60 and 75 (414 and 517 MPa) bars based on ACI 318-05 with those based on these provisions. As shown in the table, setting  $\omega = 1.0$  produces splice lengths that are approximately equal to Class B splice lengths under ACI 318-05. Because the equations in Section 2.2 are based on  $\omega = 1.0$ , the development lengths calculated with this section automatically equal Class B splice lengths under these provisions. ACI 318-05 requires that Class B splices must be used when  $A_s$  provided is less than twice  $A_s$  required or when more than 50% of the bars are spliced at one location. The result is that Class B splices are used in the majority of cases under the provisions of ACI 318-05. Because the development lengths required by these provisions are, in typical cases, longer than required by ACI 318-05, splice length and development length can remain the same under these provisions with no loss in reliability. Longer lap splices are required for tension members under Section 3.3, where a factor of 1.25 is used.

### 4.0—Development and splicing of high relative rib area deformed reinforcing bars in tension

**4.1—Development and lap splice length of deformed reinforcing bars with a high relative rib area  $R_r$ , meeting the requirements of 4.2.1 and satisfying the other criteria in 4.2, shall be permitted to be computed using the provisions of 2.0 and 3.0, respectively, with  $K'_t$  multiplied by the factor  $0.3 + 9.2R_r$ , but not greater than 1.6. For  $f'_c \leq 10,000$  psi (70 MPa),  $\psi_e = 1.2$  for epoxy-coated bars meeting the criteria in 4.2.**

**R4.1—This section is provided to help designers take advantage of high relative rib area on the tension development and lap splice length of reinforcing bars. The relative rib area is expressed as**

$$R_r = \frac{A_r}{\pi d_b s_r}$$

where

$A_r$  = projected rib area normal to reinforcing bar axis, in.<sup>2</sup> (mm<sup>2</sup>); and

$s_r$  = average center-to-center rib spacing, in. (mm).

The variables  $A_r$  and  $s_r$  are illustrated in Fig. R4.1. The figure includes expressions for the approximate values of  $A_r$  and  $R_r$ .

The value of  $\psi_e = 1.2$  for  $f'_c \leq 10,000$  psi (70 MPa) is based on the less detrimental effect of epoxy coating on the bond strength of high relative rib area bars compared with its effect on

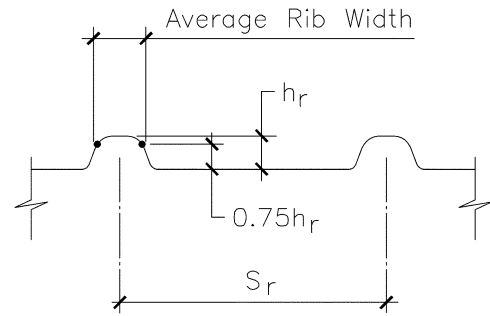


Fig. R4.2—Definition of average rib width.<sup>10</sup>

the bond strength of conventional reinforcement.<sup>2,3,6</sup> For  $f'_c > 10,000$  psi (70 MPa),  $\psi_e = 1.5$ , as it is for conventional bars.

To employ this section, the specification for reinforcing bars, ASTM A 615/A 615M,<sup>11</sup> should be modified by the supplementary requirements imposed by the Recommended Supplement to ASTM A 615/A 615M for High Relative Area Bars that appears as Appendix A of this document. With modifications to the section reference numbers, this supplement can also be adapted for use with ASTM A 706/A 706M reinforcing bars. Background information on high relative reinforcement can be obtained from ACI 408.3-01/408.3R-01,<sup>10</sup> as well as References 2, 3, 6, 7, and 12.

**4.2—High relative rib area deformed reinforcing bars shall conform to the requirements of 4.2.1 through 4.2.4.**

**4.2.1—The relative rib area  $R_r$  shall be at least 0.10, but not larger than 0.14 for use in calculating  $K'_t$  in 4.1.**

**4.2.2—The rib face shall be placed at an angle of 45 to 65 degrees, inclusive, with respect to the axis of the bar. Ribs shall not cross. Use of X-patterns or diamond patterns for ribs is not permitted.**

**4.2.3—The rib spacing shall be at least 0.44 of the nominal diameter  $d_b$  of the reinforcing bar.**

**4.2.4—The average rib width shall be less than or equal to one-third of the average rib spacing.**

**R4.2—A high relative rib area bar is defined as a reinforcing bar with  $R_r$  greater than or equal to 0.10, while conventional reinforcement typically has relative rib areas of 0.06 to 0.085. Based on available experimental results, the use of these provisions is limited to a maximum value of  $R_r = 0.14$ . Furthermore, consistent with the smallest spacing used in tests, the rib spacing  $s_r$  may not be less than 44% of the nominal bar diameter as indicated in 4.2.3. A lower limit on the width of the concrete between ribs is indirectly prescribed in 4.2.4 to avoid having a reduction in bond capacity due to a local shear failure of the concrete between the ribs. The variables in 4.2.4 are illustrated in Fig. R4.2. For calculating the average rib width, the width at 0.75 of the rib height, as illustrated in Fig. R4.2, was chosen for use in the recommended supplement to ASTM A 615 due to the possible presence of rounded corners on the ribs.**

Reinforcing bars with X or diamond deformation patterns may not be used on high  $R_r$  bars because their bond properties are markedly lower than bars with parallel ribs. The bamboo pattern for ribs (ribs oriented at 90 degrees to the bar axis) are also excluded by the angle restrictions adopted on high  $R_r$  bars because of problems associated with the bending of conventional bars with this rib orientation.

**4.3—The limitations in 4.2 shall be waived where it is demonstrated by tests that other deformation configurations provide bond strengths commensurate with 4.1.**



**R4.3**—Guidance on appropriate bond tests is provided in Reference 2.

## REFERENCES

1. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (318R-05)," American Concrete Institute, Farmington Hills, Mich., 2005, 430 pp.
2. ACI Committee 408, "Bond and Development of Straight Reinforcement in Tension (ACI 408R-03)," American Concrete Institute, Farmington Hills, Mich., 2003, 49 pp.
3. Zuo, J., and Darwin, D., "Splice Strength of Conventional and High Relative Rib Area Bars in Normal and High Strength Concrete," *ACI Structural Journal*, V. 97, No. 4, July-Aug. 2000, pp. 630-641.
4. Grundhoffer, T.; Mendis, P. A.; French, C. W.; and Leon, R., "Bond of Epoxy-Coated Reinforcement in Normal and High-Strength Concrete," *Bond and Development of Reinforcement: A Tribute to Dr. Peter Gergely*, SP-180, Roberto Leon, ed., American Concrete Institute, Farmington Hills, Mich., 1998, pp. 261-297.
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9. Sivakumar, B.; Gergely, P.; and White, R. N., "Suggestions for the Design of R/C Lapped Splices for Seismic Loading," *Concrete International*, V. 5, No. 2, Feb. 1983, pp. 46-50.
10. ACI Committee 408, "Splice and Development Length of High Relative Rib Area Reinforcing Bars in Tension (ACI 408.3-01) and Commentary (408.3R-01)," American Concrete Institute, Farmington Hills, Mich., 2001, 6 pp.
11. ASTM A 615/A 615M-04b, "Specification for Deformed and Plain Carbon-Steel Bars for Concrete Reinforcement," ASTM International, West Conshohocken, Pa., 2004, 5 pp.
12. Darwin, D., and Graham, E. K., "Effect of Deformation Height and Spacing on Bond Strength of Reinforcing Bars," *ACI Structural Journal*, V. 90, No. 6, Nov.-Dec. 1993, pp. 646-657.

## APPENDIX A—RECOMMENDED SUPPLEMENT TO ASTM A 615/A 615M<sup>11</sup> FOR HIGH RELATIVE RIB AREA BARS

The following supplementary requirements shall apply only when specified in the purchase order or contract.

### S.1—Requirements for deformations

**S.1.1** The deformations on high relative rib area bars shall meet all requirements in Section 7.

**S.1.2** In addition, the relative rib area (as defined in S.2.1) shall meet the following requirements:

**S.1.2.1** The relative rib area shall be at least 0.10, but no larger than 0.14;

**S.1.2.2** The ribs shall be oriented at an angle of 45 to 65 degrees inclusive with respect to the axis of the bar. Ribs shall not cross. Use of X-patterns or diamond patterns for ribs is not permitted;

**S.1.2.3** The rib spacing shall be at least 0.44 of the nominal diameter  $d_b$  of the reinforcing bar; and

**S.1.2.4** The average rib width shall be less than or equal to one-third of the average rib spacing.

### S.2—Relative rib area

**S.2.1** The relative rib area  $R_r$  is the ratio of the projected rib area normal to the bar axis to product of the nominal bar perimeter and average center-to-center rib spacing. The value of  $R_r$  should be specified by the purchaser.

**S.2.2** For bars that meet the requirements of S.1.2, it shall be permitted to calculate  $R_r$  using Eq. (S-1).

$$R_r = \frac{h_r}{s_r} \left( 1 - \frac{\sum \text{gaps}}{p} \right) \quad (\text{S-1})$$

where

$h_r$  = average height of deformations (measured according to S.2.3), in. (mm)

$s_r$  = average spacing of deformations, in. (mm)

$\sum \text{gaps}$  = sum of the gaps between ends of deformations as defined in Section 7.4, plus the width of any continuous longitudinal lines used to represent the grade of the bar, multiplied by the ratio of the height of the line to  $h_r$ , in. (mm)

$p$  = nominal perimeter of the bar, in. (mm) (Table 1)

**S.2.3** The average height of deformations shall be determined from measurements made on not less than two typical deformations on each side of the bar. Determinations shall be based on five measurements per deformation, one at the center of the overall length, two at the ends of the overall length, and two located halfway between the center and the ends. The measurements at the ends of the overall length shall be averaged to obtain a single value and that value shall be combined with the other three measurements to obtain the average rib height  $h_r$ . Deformation height shall be measured using a depth gauge with a knife edge support that spans not more than two adjacent ribs. Alternatively, it shall be permitted to use a knife edge that spans more than two adjacent ribs, in which case the average rib height shall be multiplied by 0.95 prior to use in Eq. (S-1).

**S.2.4** The average rib width shall be determined from measurements made on not less than two typical deformations on each side of the bar. Determinations shall be based on three measurements per deformation, one at the center and one at each end. The measurements shall be taken at three-quarters of the rib height at each location. The average of the measurements at the ends shall be averaged with the center measurement to obtain a value for the one side of the deformation.

*Note S.2—A knife edge is required to allow measurements to be made at the ends of the overall length of deformations, usually adjacent to a longitudinal rib. The calculation of  $h_r$  is based on a knife edge that spans only two ribs because measurements made with a longer knife edge result in unrealistically high average rib heights and an overestimate of the relative rib area for some bars. When a longer knife edge is used,  $h_r$  is reduced by 5%.*

### S.3—Type of steel

**S.3.1** All bars produced to these supplementary requirements shall be identified by the letter H, in place of the letter S specified in 20.3.3, indicating that the bar was produced to meet both the specification and these supplementary requirements.