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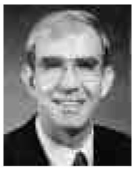
Development Length Criteria for Conventional and High Relative Rib Area Reinforcing Bars




by David Darwin, Jun Zuo, Michael L. Tholen, and Emmanuel K. Idun

Statistical analyses of 133 splice and development specimens in which the bars are not confined by transverse reinforcement and 166 specimens in which the bars are confined by transverse reinforcement are used to develop an expression for the bond force at failure as a function of concrete strength, cover, bar spacing, development/splice length, transverse reinforcement, and the geometric properties of the developed/spliced bars. Results are used to formulate design criteria that incorporate a reliability-based strength reduction (φ) factor that allows the calculation of a single value for both development and splice length for given material properties and member geometry.

As with earlier studies, the analyses demonstrate that the relationship between bond force and development or splice length l_d is linear but not proportional. Thus, to increase the bond force (or bar stress) by a given percentage requires more than the percentage increase in l_d. $f_c'^{1/2}$ does not provide an accurate representation of the effect of concrete strength on bond strength over the full range of concrete strengths in use today; development/splice strengths are underestimated for low-strength concretes and overestimated for high-strength concretes. $f_c'^{1/4}$ provides an accurate representation of the effect of concrete strength on bond strength for concretes with compressive strengths between 2500 and 16,000 psi (17 and 110 MPa). The most accurate representation of the effect of transverse reinforcement on bond strength obtained in the current analysis includes parameters that account for the number of transverse reinforcing bars that cross the developed/spliced bar, the area of the transverse reinforcement, the number of bars developed or spliced at one location, the relative rib area of the developed/spliced bar, and the size of the developed/spliced bar. The yield strength of transverse reinforcement does not play a role in the effectiveness of the transverse reinforcement in improving development/splice strength. Depending on the design expression selected, for conventional and high relative rib area bars that are not confined by transverse reinforcement, development lengths average 2 to 14 percent higher and splice lengths 12 to 22 percent lower than those obtained using ACI 318-95. For conventional reinforcing bars confined by transverse reinforcement, development lengths average 5 percent lower to 16 percent higher than those obtained using ACI 318-95, while splice lengths average 11 to 27 percent lower than those obtained using ACI 318-95. For high relative rib area reinforcing bars confined by transverse reinforcement, development lengths average 3 to 17 percent lower than those obtained using ACI 318-95, while splice lengths average 25 to 36 percent lower than those obtained using ACI 318-95. When confined by transverse reinforcement, high relative rib area bars require development and splice lengths that are 13 to 16 percent lower than required by conventional bars.

Keywords: bond (concrete to reinforcement); bridge specifications; building codes; deformed reinforcement; development; lap connections; reinforcing steels; relative rib area; reliability; splicing; structural engineering.

The provisions in Chapter 12 of the 1995 ACI Building Code (ACI 318-95) will make the design process easier and reflect development and splice strength better than any previous code procedures. The new expressions are based, in part, on a statistical analysis carried out over 20 years ago (Orangun, Jirsa, and Breen 1975) and on recommendations based on that analysis provided by ACI Committee 408 (1990). As with previous versions of the ACI Code, the calculated development/splice lengths are proportional to the bar stress (the actual relationship is linear but not proportional), and most splice lengths are 30 percent greater than the corresponding development lengths.

Over the past 20 years, additional data has become available, and analyses of the expanded database (presented in this paper) have exposed a number of shortcomings in the ability of both the code expressions and the original statistically-based expressions to accurately represent the development and splice strength of reinforcing bars, as used in current practice. Specifically, the analyses demonstrate that the square root of the concrete compressive strength f_c' does not accurately characterize the effect of concrete strength on bond strength for the full range of concrete strengths in use today, and the yield strength of transverse reinforcement f_{yt} plays no measurable role in the contribution of confining steel to bond strength. In addition, the study by Orangun et al. (1975, 1977) and a more recent study by Darwin, McCabe, Idun, and Schoenekase (1992a, 1992b) have the drawback of inadvertently including top-cast and side-cast bar specimens in analyses representing bottom-cast reinforcement. Only bottom-cast bars are considered in the current study.

The current analyses were carried out in conjunction with a large-scale experimental study to improve the development characteristics of reinforcing bars (Darwin and Graham

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***David Darwin**, FACI, is the Deane E. Ackers Professor of Civil Engineering and Director of the Structural Engineering and Materials Laboratory at the University of Kansas. He is a past member of the Board of Direction and the Technical Activities Committee and is a past-president of the ACI Kansas Chapter. Darwin is Chairman of the Publications Committee, past-chairman of the Concrete Research Council, and a member and past-chairman of ACI Committee 224, Cracking. He is also a member of ACI Committees 408, Bond and Development of Reinforcement; 446, Fracture Mechanics; and ACI-ASCE Committees 445, Shear and Torsion, and 447, Finite Element Analysis of Reinforced Concrete Structures. He is a recipient of the Arthur R. Anderson Award and the ACI Structural Research Award.*

*ACI member **Jun Zuo** is a graduate research assistant in civil engineering at the University of Kansas. He obtained his BS in architectural engineering from Tongji University in China and his MS in civil engineering from the University of Kansas. He is currently pursuing his PhD degree.*

*ACI member **Michael L. Tholen** is a graduate research assistant in civil engineering at the University of Kansas, where he holds a BS in architectural engineering and an MS in civil engineering. He is currently pursuing his PhD degree.*

*ACI member **Emmanuel K. Idun** is a lead structural engineer with Constructive Engineering Design, Kansas City, Missouri. He holds a BScEng in civil engineering and an MPhil in structures from the University of Science and Technology in Ghana, and a PhD from the University of Kansas.*

1993a, 1993b, Darwin, Tholen, Idun, and Zuo 1995a, 1996a) and have several advantages over the earlier studies: 1) the database is larger (Chinn et al. 1955, Chamberlin 1956, 1958, Mathey and Watstein 1961, Ferguson and Thompson 1965, Ferguson and Breen 1965, Thompson et al. 1975, Zekany et al. 1981, Choi et al. 1990, 1991, DeVries et al. 1991, Hester et al. 1991, 1993, Rezansoff et al. 1991, 1993, Azizinamini et al. 1993, 1995, Darwin et al. 1995a, 1996a), including 133 splice and development specimens in which the bars are not confined by transverse reinforcement and 166 specimens in which the bars are confined by transverse reinforcement; 2) the concrete strengths cover a broader range than used in the earlier studies; and 3) data includes bars with a wide range of relative rib area (ratio of bearing area of ribs to shearing area between ribs) R_r, a parameter that has been demonstrated to significantly affect the added bond strength provided by transverse reinforcement (Darwin and Graham 1993a, 1993b, Darwin et al. 1995a, 1996a).

This paper describes the development of a statistically-based expression that accurately represents the development and splice strength of reinforcing bars, both with and without confining reinforcement, for values of f'_c between 2500 and 16,000 psi (17 and 110 MPa). In addition to transverse reinforcement and concrete strength, the expression takes into account cover, bar spacing, development/splice length, and the geometric properties of the developed/spliced bars. The expression is used to formulate design criteria that incorporate a reliability-based strength reduction (ϕ) factor (Darwin, Idun, Zuo, and Tholen 1995c, 1996b) that allows the calculation of a single value for both splice and development length for given material properties and member geometry. Compared to current design practice (ACI 318-95, AASHTO Highway 1992), the new design criteria permit major reductions in the development lengths of high relative rib area bars confined by transverse reinforcement and in the splice lengths of conventional and high relative rib area bars under all conditions of confinement. Additional details of the study are presented by Darwin, Zuo, Tholen and Idun (1995b).

OVERVIEW

The statistical analyses and development of design criteria that are described in this paper are based on a model in which the maximum bond force in a developed or spliced bar T_b is expressed as the sum of a "concrete contribution" T_c, which is a function of concrete strength, member geometry, and bar size, and a "steel contribution" T_s, which is a function of concrete strength, the geometric properties of the developed/spliced bar, and the geometry of the confining reinforcement in the development/splice region

$$T_b = T_c + T_s \tag{1}$$

Eq. (1) serves as the basis of the analysis that, when complete, is used to formulate design expressions that are used to calculate development/splice length l_d.

The calculation of the concrete contribution T_c builds on earlier work (Orangun et al. 1975, 1977, Darwin et al. 1992a, 1992b). The analysis initially proceeds by determining the best statistical match between the total bond force for bars not confined by transverse reinforcement $T_c = A_b f_s$, in which A_b = bar area and f_s = bar stress at development or splice failure, and the product of l_d, the development or splice length, and $c_m + 0.5\ d_b$, the smaller of the cover to the center of the bar ($c_b + 0.5\ d_b$) or half the center-to-center bar spacing ($c_s + 0.5\ d_b$), in which c_b = cover, c_s = one-half of the clear spacing between bars, and d_b = bar diameter. Next, adjustments are made to take into account the fact that bond strength increases with respect to the product $l_d(c_m + 0.5\ d_b)$ as the difference between c_b and c_s increases.

The initial analysis is carried out using (as is traditional) $f'^{1/2}_c$ to represent the effect of concrete strength on bond strength. The resulting expression is tested for f'_c between 2610 and 15,120 psi (18 and 104 MPa), and the power of f'_c is adjusted to provide an improved representation for bond strength. The new expression for T_c is then used to calculate the steel contribution T_s in development/splice tests for members containing confining reinforcement. This is done by subtracting the calculated value of the concrete contribution from the experimental bond force T_b

$$T_s = T_b - T_c \tag{2}$$

T_s is correlated with the concrete strength, the geometric properties of the transverse reinforcement, and the geometric properties of the developed/spliced bars to obtain an accurate representation of the increase in bond strength provided by the confining steel. The evaluation includes the establishment of limits within which the expressions give conservative predictions of strength.

The resulting expressions for bond force for developed/spliced bars, both with and without confining reinforcement, are then combined with a reliability-based strength reduction (ϕ) factor (Darwin et al. 1995c, 1996b) to obtain design expressions for l_d. The expressions include the effect of relative rib area R_r, and thus, can be used to take advantage of the increased bond strength obtainable with high R_r bars. The development and splice lengths obtained with the new expressions are then compared to those obtained using ACI 318-95.

Test specimens used in the analyses are limited to splice and development specimens for which concrete properties are characterized by the compressive strength of standard cylinders (ASTM C 39).

EXPRESSIONS FOR DEVELOPMENT/SPLICE STRENGTH

Bars without confining reinforcement

The work reported herein represents the final results of a series of analyses using 133 development and splice specimens containing bottom-cast bars.

Using $f'_c{}^{1/2}$ to represent the effect of concrete compressive strength on bond strength produces the following expression for total bond force for bars not confined by transverse reinforcement

$$\frac{T_c}{f'_c{}^{1/2}} = \frac{A_b f_s}{f'_c{}^{1/2}} = [8.76 l_d (c_m + 0.5 d_b) + 187 A_b]\left(0.14\frac{c_M}{c_m} + 0.86\right) \quad (3)$$

in which

c_m, c_M = minimum and maximum value of c_s or c_b ($c_M/c_m \leq 3.5$), in in.

c_s = min (c_{si} + 0.25 in., c_{so}), in.

c_{si} = one-half of clear spacing between bars, in.

c_{so}, c_b = side cover and bottom cover of reinforcing bars, in.

T_c is in lb, A_b is in in.^{2}, and f_s, f'_c, and $f'_c{}^{1/2}$ are in psi.

Eq. (3) is obtained following the procedures of Darwin et al. (1992a, 1992b). A best-fit is obtained between $T_c/f'_c{}^{1/2}$ and the product $l_d(c_m + 0.5\, d_b)$ using a dummy variable analysis (Draper and Smith, 1981) in which the data are separated based on bar size. The results of the analysis are then used to improve the fit by including a weighted average coefficient to represent the area of the bar A_b. Unlike the earlier analysis (Darwin et al. 1992a, 1992b), the effects of the differences in c_m and c_M are evaluated after the coefficient for A_b is obtained.

The term (0.14 c_M/c_m + 0.86) is obtained based on a best-fit analysis comparing the test/prediction ratios [obtained using the term in brackets on the right side of Eq. (3) as the predicted strength] with the ratio c_M/c_m. The term takes into account the increased strength observed in the tests when $c_m \neq c_M$. When determining c_s, 0.25 in. (6 mm) is added to c_{si}, one-half of the clear spacing between the bars, because the extra 0.25 in. (6 mm) gives an improved match with the test data. The fact that the effective value of c_{si} is slightly larger than one-half of the clear spacing is likely due to the longer effective crack lengths that occur when concrete splits between the bars rather than through the cover (Darwin et al. 1992a, 1992b).

When the test results used to develop Eq. (3) are re-evaluated based on categories of concrete strength, the specimens with the lowest strength concretes produce the highest relative strengths, as shown in Fig. 1. For the categories of concrete strengths evaluated, from below 3000 to over 10,000 psi (21 to 69 MPa), the intercepts on the vertical axis decrease as the concrete strength increases. The line representing concrete with compressive strengths above 10,000 psi (69 MPa) is significantly below that of the rest of the data. The comparisons show that $f'_c{}^{1/2}$ gives a good representation for concrete strengths between 4500 and 7500 psi (31 and 52 MPa). Outside of this range, $f'_c{}^{1/2}$ does not give a good representation.

Based on this observation, a series of reanalyses were carried out to determine the power of f'_c that would minimize the spread in the data. The reanalyses showed that f'_c to the 0.24 power provided the best match. For obvious reasons of convenience, the $^1/_4$ power was selected for further analysis.

Using the $^1/_4$ power, the best-fit equation is

$$\frac{T_c}{f'_c{}^{1/4}} = \frac{A_b f_s}{f'_c{}^{1/4}} = [63 l_d (c_m + 0.5 d_b) + 2130 A_b]\left(0.1\frac{c_M}{c_m} + 0.9\right) \quad (4)$$

in which $f'_c{}^{1/4}$ is in psi.

As illustrated in Fig. 2, Eq. (4) produces significantly less scatter as a function of compressive strength than Eq. (3). The best-fit lines for all categories of concrete strength nearly coincide, with the exception of the specimens with concrete strengths in excess of 10,000 psi (69 MPa). This deviation is largely the result of the limited amount of data for development/splice tests using high-strength concrete. Two relatively low splice strengths have a dominant effect on the results for this category. If those two tests are removed, all strength categories produce nearly coincident best-fit lines (Darwin et al. 1995b).

Table 1 provides a summary of the test/prediction ratios for the 133 specimens used to develop Eq. (3) and (4). As shown in the table, the mean test/prediction ratio for the 133 specimens without transverse reinforcement is 1.00 using both the $^1/_2$ [Eq. (3)] and the $^1/_4$ [Eq. (4)] power of f'_c, with a coefficient of variation (COV) of 0.138 using the $^1/_2$ power of f'_c and a COV of 0.107 using the $^1/_4$ power. The individual comparisons are presented by Darwin et al. (1995b) and in Appendix A.*

Bars with confining reinforcement

Eq. (2) is used to determine the additional bond strength provided by transverse reinforcement T_s. The concrete contribution to bond strength T_c, given in Eq. (4), is subtracted from the experimental bond force T_b. The results for 166 specimens in which the developed/spliced bars were confined by transverse reinforcement were initially used for this analysis. During the course of the analysis, it was established that especially low strengths, with respect to any predictive equations, were exhibited by specimens with l_d/d_b < 16. Therefore, 32 specimens with l_d/d_b < 16 have been removed

*The Appendix is available in xerographic or similar form from ACI headquarters, where it will be kept permanently on file, at a charge equal to the cost of reproduction plus handling at time of request.

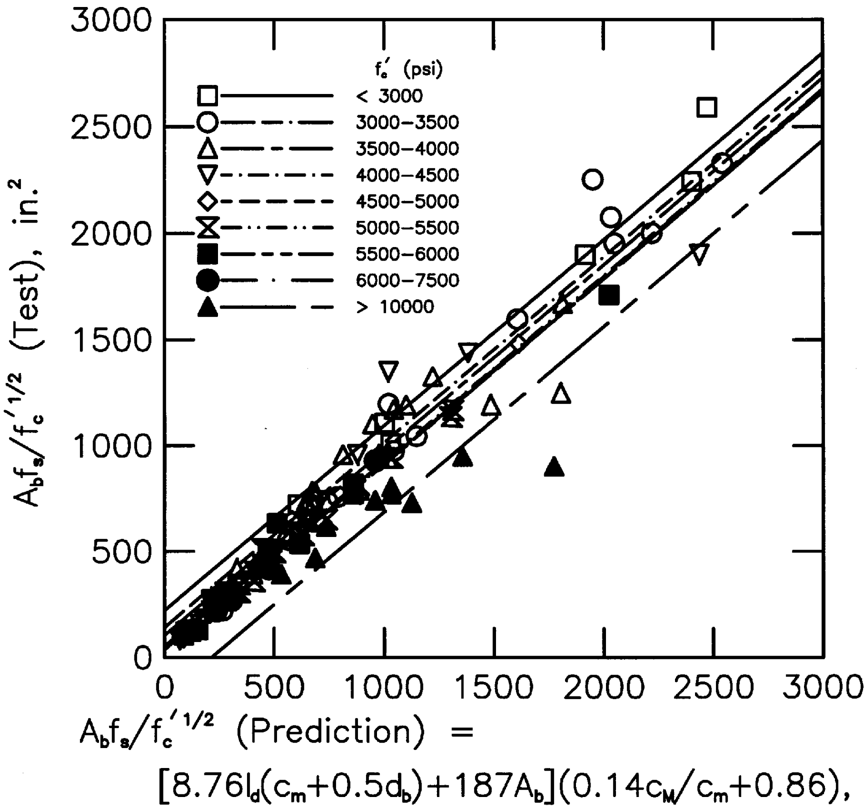

Fig. 1—Experimental bond force $T_c = A_bf_s$ normalized with respect to $f'_c{}^{1/2}$ versus predicted bond force $A_b f_s/f'_c{}^{1/2}$, as a function of concrete compressive strength for bars without confining reinforcement

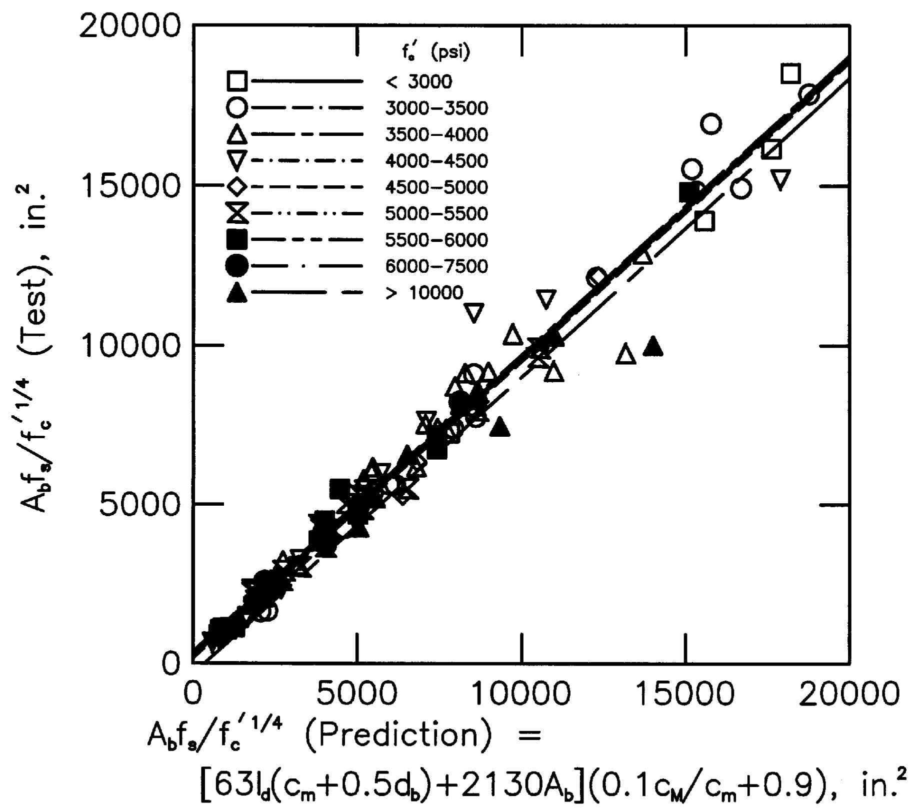

Fig. 2—Experimental bond force $T_c = A_bf_s$ normalized with respect to $f'_c{}^{1/4}$ versus predicted bond force $A_bf_s/f'_c{}^{1/4}$ as a function of concrete compressive strength for bars without confining reinforcement

from the analysis, leaving 134 specimens for the following analysis. The removal of these specimens does not hurt the overall evaluation, since members with such low values of l_d/d_b are not used in practice.

Correlations of T_s with several combinations of potential controlling parameters are evaluated. Principal among these parameters are the yield strength of the transverse reinforcement f_{yt} and the effective area of transverse reinforcement per developed/spliced bar NA_{tr}/n, in which N = the number of transverse reinforcing bars (stirrups or ties) crossing l_d; A_{tr} = area of each stirrup or tie crossing the potential plane of splitting adjacent to the reinforcement being developed or spliced, and n = number of bars being developed or spliced along the plane of splitting. The value of n is determined by the smaller of c_b or c_s. If c_b controls, the plane of splitting passes through the cover and n = 1. If c_s controls, the plane of splitting intersects all of the bars and n = the total number of bars spliced or developed at one location. Also included in the analysis are parameters t_r and t_d, representing the effects of the relative rib area and bar size, respectively, of the developed/spliced bar on T_s

$$t_r = 9.6R_r + 0.28 \tag{5}$$

$$t_d = 0.72d_b + 0.28 \tag{6}$$

Eq. (5) and (6) are based on an analysis of test results for 70 splice specimens containing No. 5, No. 8, and No. 11 (16, 25, 36-mm) bars confined by transverse reinforcement with relative rib areas R_r ranging from 0.065 to 0.14. Details of the development of Eq. (5) and (6) are presented by Darwin et al. (1995a, 1996a). For conventional reinforcement, t_r typically ranges from 0.82 to 1.11 (for R_r from 0.056 to 0.086), with an average value of 0.98 [for the average value of R_r = 0.0727 (Darwin et al. 1995b)]; t_d = 0.73, 1.00, and 1.295 for No. 5, No. 8, and No. 11 (16, 25, 36-mm) bars, respectively.

To determine the principal controlling parameters, T_s is compared to four combinations of the parameters; $NA_{tr}f_{yt}/n$, NA_{tr}/n, t_rNA_{tr}/n, and $t_rt_dNA_{tr}/n$. The first of these variables, $NA_{tr}f_{yt}/n$, is incorporated in ACI 318-95 to represent the effect of confining reinforcement on bond strength (in ACI 318-95, $N = l_d/s$, in which s = spacing of transverse reinforcement).

In carrying out the analyses, distinct differences are observed in the test results for different investigators. For example, the bond strengths obtained by Rezansoff et al. (1991, 1993) are consistently higher than those obtained by Choi et al. (1990, 1991), Hester et al. (1991, 1993), and Darwin et al. (1995a, 1996a). The differences, in all likelihood, are due to differences in concrete properties and, perhaps, testing procedures. The effect of concrete properties on bond strength is demonstrated by Darwin et al. (1995a, 1996a), who observed 35 to 45 percent changes in the effectiveness of transverse reinforcement with a change in coarse aggregate. To remove the variation caused by differences in concrete properties or other differences between test sites, the study uses a dummy variables analysis in which the data is separated based on test site and bar size.

Of the 134 specimens used in the analysis, the value of R_r is known for 85 specimens, based on measurements made on the bars or based on data provided in the original papers. For the balance of the bars, the mean values of R_r for bars of that size are used. The mean values, 0.0752 for No. 5 (16-mm) bars, 0.0748 for No. 6 (19-mm) bars, 0.0731 for No. 8 (25-mm) bars, and 0.0674 for No. 11 (36-mm) bars, are based on bar samples measured in studies dating to 1987 (Choi et al.

Table 1—Summary of test/prediction ratios for developed and spliced bars

Specimen type	Number of specimens	Power of f'_c (Eq.)	Minimum	Maximum	Mean	Standard deviation	Coefficient of variation
Without transverse reinforcement	133	1/2 [Eq. (3)] 1/4 [Eq. (4)]	0.509 0.716	1.325 1.290	1.000 1.003	0.138 0.107	0.138 0.107
Without transverse reinforcement, $f_s > f_y$	11	1/2 [Eq. (3)] 1/4 [Eq. (4)]	0.783 0.854	1.213 1.275	0.968 0.992	0.112 0.107	0.115 0.107
With transverse reinforcement	166	1/4 [Eq. (17)]	0.571	1.387	0.979	0.138	0.141
With transverse reinforcement, l_d/d_b ŠŠŠŠŠ ≥ 16	134	1/4 [Eq. (17)]	0.664	1.352	0.989	0.135	0.137
With transverse reinforcement, l_d/d_b ŠŠŠŠŠ ≥ 16, $(c + K_{tr})/d_b \leq 4$*	119†	1/4 [Eq. (17)]	0.770	1.352	1.010	0.127	0.125
With transverse reinforcement, $f_s > f_y$, l_d/d_b ŠŠŠŠŠ ≥ 16, $(c + K_{tr})/d_b \leq 4$*	20	1/4 [Eq. (17)]	0.931	1.352	1.153	0.154	0.134
With transverse reinforcement, $f_s > f_y$, l_d/d_b ŠŠŠŠŠ ≥ 16, $(c + K_{tr})/d_b \leq 4$*	99	1/4 [Eq. (17)]	0.770	1.261	0.981	0.098	0.100

*Based on $K_{tr} = 35.3\, t_rt_dA_{tr}/sn$.

†Includes two specimens with $(c + K_{tr})/d_b > 4$: a) $(c + K_{tr})/d_b = 4.004$, test/prediction = 0.843; b) $(c + K_{tr})/d_b = 4.023$, test/prediction = 0.901.

1990, 1991, Hester et al. 1991, 1993, Darwin et al. 1995a), including bar samples provided by other researchers (Rezansoff et al. 1991, 1993, Azizinamini et al. 1995). The overall average value of R_r, 0.0727, represents No. 5 and larger bars. R_r = 0.0727 is used for bar sizes other than No. 5, No. 6, No. 8, and No. 11 (16, 19, 25, 36 mm), if individual data is not available. For "metric bars" (Rezansoff et al. 1991, 1993), nominal metric sizes are converted exactly to customary units for the analysis. For the analysis, T_s is in lb, f_{yt}, f'_c and $f'^{1/4}_c$ are in psi, and A_{tr} is in in.^{2} The database includes specimens with concrete strengths between 1820 and 15,760 psi (13 and 109 MPa) and bars with relative rib areas between 0.059 and 0.14.

Based on the dummy variables analyses and using the weighted mean intercepts at $T_s/f'^{1/4}_c = 0$, the best-fit expressions for the four combinations are

$$\frac{T_s}{f'^{1/4}_c} = 26.7\frac{NA_{tr}f_{yt}}{n} + 355 \quad (7)$$

with a coefficient of determination $r^2 = 0.757$.

$$\frac{T_s}{f'^{1/4}_c} = 2391\frac{NA_{tr}}{n} + 89 \quad (8)$$

with $r^2 = 0.787$.

$$\frac{T_s}{f'^{1/4}_c} = 2093t_r\frac{NA_{tr}}{n} + 110 \quad (9)$$

with $r^2 = 0.840$.

$$\frac{T_s}{f'^{1/4}_c} = 1867t_r t_d\frac{NA_{tr}}{n} + 177 \quad (10)$$

with $r^2 = 0.839$.

The closer the coefficient of determination r^2 is to 1.0, the better the correlation between $T_s/f'^{1/4}_c$ and the selected combination of parameters. r^2 is lowest (0.757) when $NA_{tr}f_{yt}/n$ is used to represent the effect of transverse reinforcement on bond strength [Eq. (7)]. Removal of f_{yt} from the controlling variable [Eq. (8)] improves r^2 to 0.787. The fact that such an improvement would occur makes sense, since it has been demonstrated that transverse reinforcement rarely yields during a splice or development failure (Maeda et al. 1991, Sakurada et al. 1993, Azizinamini et al. 1995). The addition of t_r to the analysis [Eq. (9)], as supported by the experimental work of Darwin et al. (1995a, 1996a), improves r^2 to 0.840, while the addition of t_d [Eq. (10)], also supported by Darwin et al. (1995a, 1996a), drops r^2 slightly to 0.839. For reasons that will be clear shortly, Eq. (10) is used for the next step in the analysis.

Combining Eq. (4) and Eq. (10), replacing N by l_d/s, dropping the mean intercept of 177, and solving for the development/splice length l_d gives

$$l_d = \frac{A_b\left[\dfrac{f_s}{f'^{1/4}_c} - 2130\left(0.1\dfrac{c_M}{c_m} + 0.9\right)\right]}{63\left[(c_m + 0.5d_b)\left(0.1\dfrac{c_M}{c_m} + 0.9\right) + \dfrac{29.6t_r t_d A_{tr}}{sn}\right]} \quad (11)$$

Modifying Eq. (11) to express l_d in terms of bar diameter d_b gives

$$\frac{l_d}{d_b} = \frac{\dfrac{f_s}{f'^{1/4}_c} - 2130\left(0.1\dfrac{c_M}{c_m} + 0.9\right)}{80.2\left(\dfrac{c + K_{tr}}{d_b}\right)} \quad (12)$$

in which $c = (c_m + 0.5\ d_b)(0.1\ c_M/c_m + 0.9)$ and $K_{tr} = 29.6\ t_r t_d A_{tr}/sn$.

$(c + K_{tr})/d_b$ in the denominator of Eq. (12) is a measure of the assistance provided by concrete cover, bar spacing, and transverse reinforcement (ACI 318-95), increases that result in an increase in bond strength. Increases in $(c + K_{tr})/d_b$, however, will eventually cause the mode of bond failure to switch from splitting to pullout, with bond strength limited by the strength of the concrete between the ribs of the bar rather than the clamping forces provided by surrounding concrete and steel. When this happens, bond strengths will drop in relation to the predicted strength.

Test/prediction ratios, based on the sum of Eq. (4) and (10), are compared with $(c + K_{tr})/d_b$ for the 134 tests with $l_d/d_b \geq 16$ in Fig. 3. The figure shows that the test/prediction ratios are consistently below 1.0 for values of $(c + K_{tr})/d_b > 3.75$. Based on this observation, a reanalysis was carried out using specimens with $(c + K_{tr})/d_b \leq 3.75$.

Based on the dummy variables analysis for the remaining 119 specimens and using the weighted mean intercepts at $T_s/f'^{1/4}_c = 0$, the best-fit expressions for the four combinations are

$$\frac{T_s}{f'^{1/4}_c} = 30.3\frac{NA_{tr}f_{yt}}{n} + 430 \quad (13)$$

with $r^2 = 0.758$.

$$\frac{T_s}{f'^{1/4}_c} = 2521\frac{NA_{tr}}{n} + 148 \quad (14)$$

with $r^2 = 0.783$.

$$\frac{T_s}{f'^{1/4}_c} = 2412t_r\frac{NA_{tr}}{n} + 71 \quad (15)$$

with $r^2 = 0.853$.

$$\frac{T_s}{f'^{1/4}_c} = 2226t_r t_d\frac{NA_{tr}}{n} + 66 \quad (16)$$

with $r^2 = 0.857$

In this case, $t_r t_d NA_{tr}/n$ [Eq. (16)] provides the best coefficient of determination and the lowest intercept. Combining Eq. (16) with Eq. (4) gives the final expression for T_b

$$\frac{T_b}{f'^{1/4}_c} = \frac{T_c + T_s}{f'^{1/4}_c} = \frac{A_b f_s}{f'^{1/4}_c} = [63 l_d (c_m + 0.5 d_b) \qquad (17)$$
$$+ 2130 A_b]\left(0.1\frac{c_M}{c_m} + 0.9\right) + 2226 t_r t_d \frac{NA_{tr}}{n} + 66$$

Dropping the intercept 66 and solving for l_d in terms of A_b and d_b gives, respectively,

$$l_d = \frac{A_b\left[\frac{f_s}{f'^{1/4}_c} - 2130\left(0.1\frac{c_M}{c_m} + 0.9\right)\right]}{63\left[(c_m + 0.5 d_b)\left(0.1\frac{c_M}{c_m} + 0.9\right) + \frac{35.3 t_r t_d A_{tr}}{sn}\right]} \qquad (18)$$

$$\frac{l_d}{d_b} = \frac{\frac{f_s}{f'^{1/4}_c} - 2130\left(0.1\frac{c_M}{c_m} + 0.9\right)}{80.2\left(\frac{c + K_{tr}}{d_b}\right)} \qquad (19)$$

in which $c = (c_m + 0.5\ d_b)(0.1\ c_M/c_m + 0.9)$ and $K_{tr} = 35.3\ t_r\ t_d A_{tr}/sn$. Eq. (19) and (12) are identical, except for the coefficient in K_{tr}.

A reanalysis of the data versus $(c + K_{tr})/d_b$ using Eq. (17) and the new definition of K_{tr} is shown in Fig. 4, illustrating that Eq. (17) through (19) provide accurate predictions for specimens with $(c + K_{tr})/d_b \leq 4.0$. A summary of the test/prediction ratios for all 166 specimens with transverse reinforcement in the database (c/d_b = 1.33 to 4.46, K_{tr}/d_b = 0.12 to 3.24) are presented in Table 1. For the 119 specimens used to develop Eq. (17) (c/d_b = 1.33 to 2.64, K_{tr}/d_b = 0.12 to 2.55), the mean test/prediction ratio is 1.01, with a COV of 0.125; two of the specimens have $(c + K_{tr})/d_b > 4.0$ (see Table 1). A comparison of the test results with the values predicted using Eq. (17) for the 117 specimens with $l_d/d_b \geq 16$ and $(c + K_{tr})/d_b \leq 4.0$ (using $K_{tr} = 35.3\ t_r t_d A_{tr}/sn$) is shown in Fig. 5 (for completeness, it is noted that c/d_b ranges from 1.33 to 3.44 for the specimens without confining reinforcement summarized in 1). Data on the individual comparisons is presented by Darwin et al. (1995b) and in Appendix A.*

Effect of bar stress on development/splice strength

Concern has been expressed that yielding of developed/spliced bars will result in a reduction in bond strength (Orangun et al. 1975, Harajli 1994). An evaluation of the test results used in the current study shows that the concern is unwarranted.

Of the 133 test specimens without confining reinforcement, bars yielded in 11 specimens prior to bond failure. As shown in Table 1, the mean test/prediction ratio based on

*The Appendix is available in xerographic or similar form from ACI headquarters, where it will be kept permanently on file, at a charge equal to the cost of reproduction plus handling at time of request.

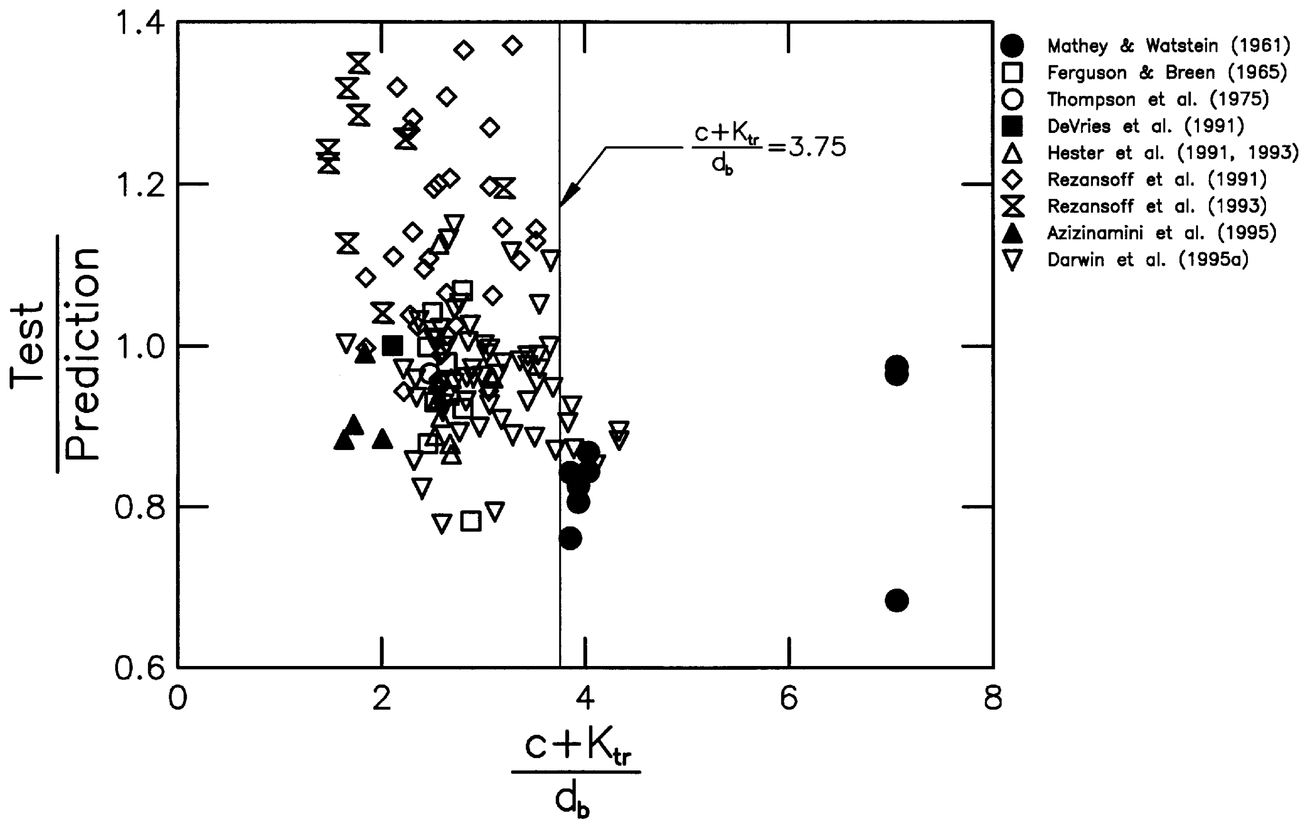

Fig. 3—Test/prediction ratio versus $(c + K_{tr})/d_b$ for 134 beams with $l_d/d_b \geq 16$ ($K_{tr} = 29.6\ t_r t_d A_{tr}/sn$)

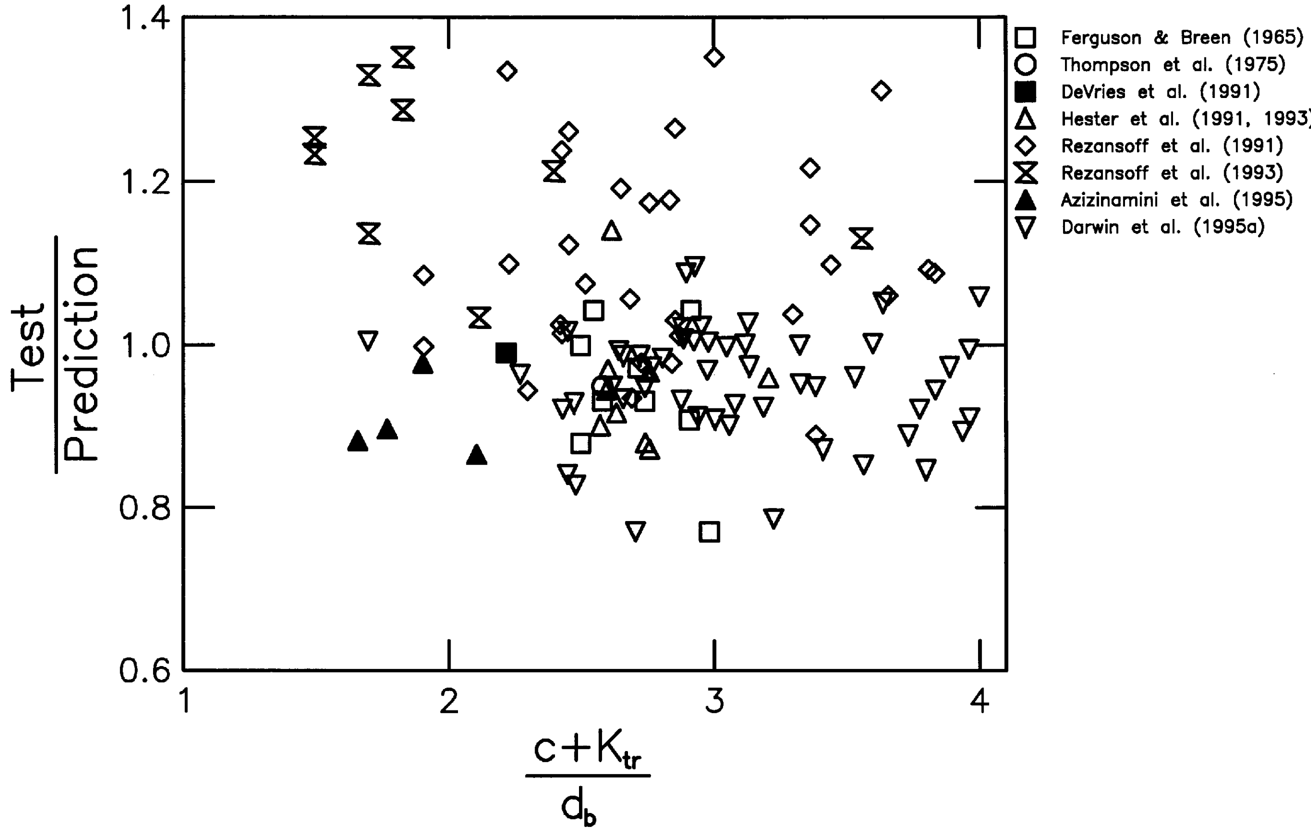

Fig. 4—Test/prediction ratio versus $(c + K_{tr})/d_b$ for 117 beams with $l_d/d_b \geq 16$ and $(c + K_{tr})/d_b \leq 4$ ($K_{tr} = 35.3\ t_r t_d A_{tr}/sn$)

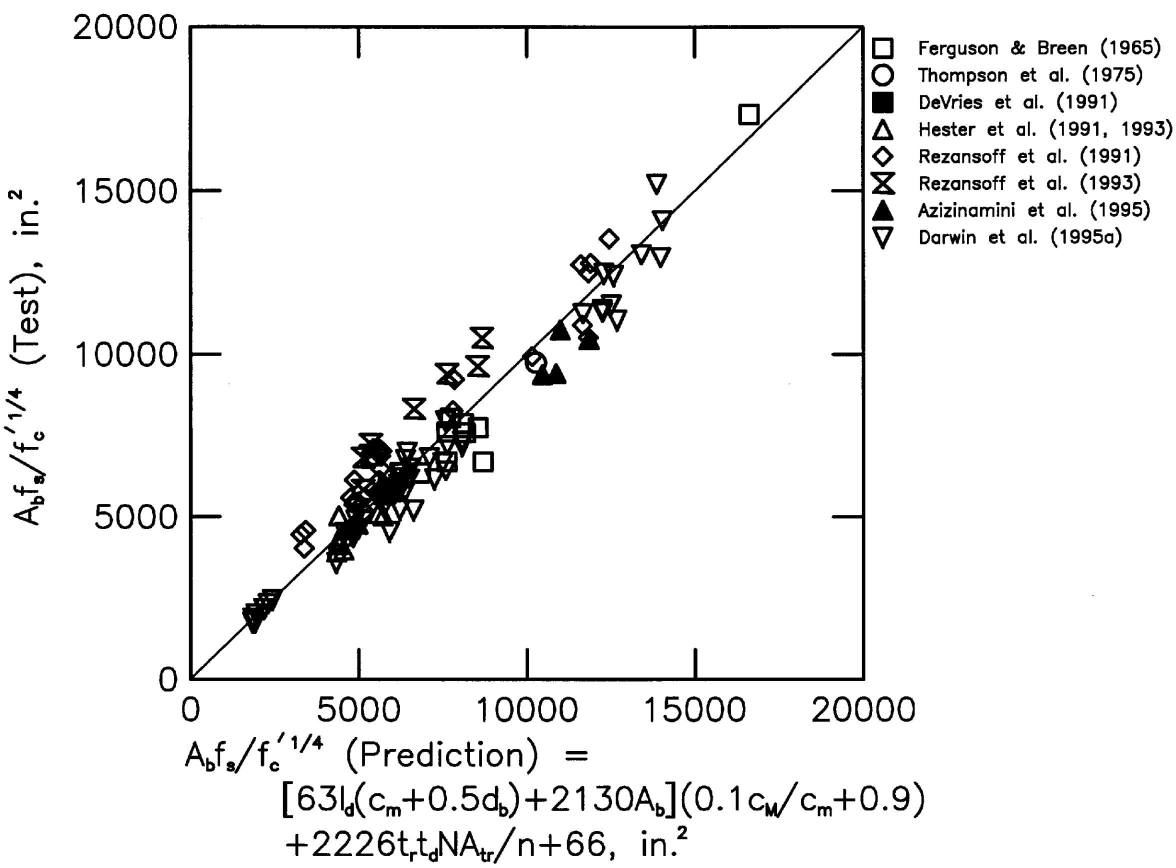

Fig. 5—Experimental bond force $T_b = A_b f_s$ normalized with respect to $f'^{1/4}_c$ versus predicted bond force $A_b f_s / f'^{1/4}_c$ for bars with confining reinforcement

Eq. (4) for the 11 tests is 0.99, with a COV of 0.107, comparing favorably to the mean of 1.00 and COV of 0.107 for the full set of data. Of the 119 bars used to develop Eq. (17), bars yielded in 20 specimens prior to bond failure. For those tests, the mean test/prediction ratio is 1.15, with a COV of 0.134, comparing very favorably with the mean of 1.01 and COV of 0.125 for the full set of 119 specimens. For the 99 tests with bars confined by transverse reinforcement that did not yield, the mean test/prediction ratio using Eq. (17) is 0.98, with a COV of 0.100.

Overall, the data indicates that, if the development/splice length is long enough to cause the bar to yield, yielding has no effect on the bond strength of bars not confined by transverse reinforcement, and results in an increase in bond strength for bars that are confined by transverse reinforcement. The increase for bars with confining reinforcement may result from a more uniform state of bond stress along the length of the bar due to greater slip that accompanies yielding. This greater slip mobilizes clamping stresses in the transverse reinforcement along a greater length of the bar.

DESIGN EXPRESSIONS FOR DEVELOPMENT/ SPLICE LENGTH

Strength reduction (ϕ) factor

Eq. (17) through (19) serve as the basis for design expressions for development/splice length. Eq. (18) and (19) cannot be used directly in design to calculate l_d because they are based on the best-fit (average) expression, Eq. (17). If used as presented, bond strength would be below the value predicted by Eq. (17) 50 percent of the time. Procedures exist, however, for insuring an adequate level of safety through the selection of a strength reduction factor (ϕ) based on the desired level of reliability.

Following the procedures of Ellingwood, Galambos, MacGregor, and Cornell (1980), Mirza and MacGregor (1986), and Lundberg (1993), a (ϕ) factor of 0.9 for development and splice strength has been obtained using a reliability index β of 3.5 (Darwin et al. 1995c, 1996b). This gives an overall probability of bond failure equal to about one-fifth of the probability of a flexural failure, for which β = 3.0 is normally obtained (Ellingwood et al. 1980). ϕ = 0.9 is obtained using Eq. (17) without the final term 66 as the design strength and Eq. (17) with the final term (if transverse reinforcement is used) as the predicted strength. Additional simplifications of Eq. (17), setting $c_M = c_m$ and dropping 0.25 in. from the definition of c_s, produce higher values of ϕ (Darwin et al. 1995c, 1996b).

ϕ = 0.9 for bond is applied in addition to the ϕ factor for the main load effect (e.g., 0.9 for flexure or 0.7 for tied columns) that is used to select the area and strength of the steel. Therefore, the total ϕ factor against a primary mode of failure in bond is the product of 0.9 and the ϕ factor for the main load effect.

In addition to allowing the selection of a desired relative probability of failure, using a reliability-based ϕ factor provides another important benefit. Since 87 percent of the tests in the database used to calculate ϕ are splice tests in which all of the bars are spliced at one location (a Class B splice in ACI 318-95 and a Class C splice in AASHTO Highway 1992), ϕ = 0.9 and Eq. (17) through (19) are already calibrated based on splice strength. Therefore, values of l_d calculated using ϕ = 0.9 apply directly to spliced bars, removing the requirement to multiply development length by 1.3 to obtain the length of a Class B splice (ACI 318-95) or by 1.7 to obtain the length of a Class C splice (AASHTO Highway 1992).

The process of obtaining the design expressions that are presented in the following starts with the incorporation of ϕ on the right side of Eq. (17) (without the final term 66) and the substitution of the bar yield strength f_y for f_s on the left side

$$\frac{A_b f_y}{f_c'^{1/4}} = \phi\left\{[63 l_d (c_m + 0.5 d_b) + 2130 A_b] \left(0.1\frac{c_M}{c_m} + 0.9\right) + 2226 t_r t_d \frac{N A_{tr}}{n}\right\} \tag{20}$$

Design expressions

Using the formulation shown in Eq. (20), a detailed design expression in the form of Eq. (19) becomes

$$\frac{l_d}{d_b} = \frac{\dfrac{f_y}{\phi f_c'^{1/4}} - 2130\left(0.1\dfrac{c_M}{c_m} + 0.9\right)}{80.2\left(\dfrac{c + K_{tr}}{d_b}\right)} \tag{21}$$

in which

$c = (c_m + 0.5\, d_b)(0.1\, c_M / c_m + 0.9)$ and c_m, c_M, c_s, c_{si}, c_{so}, and c_b are defined following Eq. (3).

$K_{tr} = K_{tr}(\text{conv.}) = 34.5\, t_d A_{tr}/sn = 34.5\,(0.72\, d_b + 0.28)\, A_{tr}/sn$ for conventional bars (average $R_r = 0.0727$)

$K_{tr} = K_{tr}(\text{new}) = 53\, t_d A_{tr}/sn = 53\,(0.72\, d_b + 0.28)\, A_{tr}/sn$ for high relative rib area bars (average $R_r = 0.1275$)

$(c + K_{tr})/d_b \le 4.0$

Incorporating ϕ = 0.9 into Eq. (21) and conservatively rounding the coefficients gives

$$\frac{l_d}{d_b} = \frac{\dfrac{f_y}{f_c'^{1/4}} - 1900\left(0.1\dfrac{c_M}{c_m} + 0.9\right)}{72\left(\dfrac{c + K_{tr}}{d_b}\right)} \tag{22}$$

Eq. (22) is the prototype for design equations based on Eq. (20). Different degrees of simplification are possible, depending on the application and the level of simplification desired.

One such simplification can be obtained by setting $c_M/c_m = 1$

$$\frac{l_d}{d_b} = \frac{\dfrac{f_y}{f_c'^{1/4}} - 1900}{72\left(\dfrac{c + K_{tr}}{d_b}\right)} \tag{23}$$

in which $c = (c_m + 0.5\, d_b)$.

In applying Eq. (23) to design, it would seem prudent to change the definition of c to the smaller of the cover to the center of the bar or one-half of the center-to-center bar spacing. The only change that this entails is dropping 0.25 in. from the definition of c_s that follows Eq. (3). The definitions of K_{tr} following Eq. (21) remain unchanged.

Following the lead of ACI 318-95, an alternate simplification of Eq. (22), for the case in which the clear spacing between bars being developed or spliced is not less than 2 d_b and the cover is not less than d_b [i.e., $(c + K_{tr})/d_b \geq 1.5$], is obtained by setting $(c + K_{tr})/d_b = 1.5$.

This gives

$$\frac{l_d}{d_b} = \frac{\dfrac{f_y}{f'^{\,1/4}_c} - 1900}{108} \qquad (24)$$

Since, except for shells, the minimum cover c_b for cast-in-place concrete is 0.75 in. (19 mm) and the minimum clear spacing 2 c_{si} is 1 in. (25 mm) (ACI 318-95), Eq. (24) provides the maximum value of l_d for No. 6 and smaller bars.

For bars with a cover not less than d_b and a clear spacing not less than 7 d_b (principally slabs), Eq. (22) can be conservatively simplified to

$$\frac{l_d}{d_b} = \frac{\dfrac{f_y}{f'^{\,1/4}_c} - 1900}{135} \qquad (25)$$

l_d from Eq. (25) is 80 percent of l_d calculated using Eq. (24). Because of the simplified format, neither Eq. (24) nor Eq. (25) takes advantage of the higher value of K_{tr} provided by high relative rib area bars. Like the simplified format in ACI 318-95 (discussed in the next section), each of the two equations provides a single value of l_d/d_b for each combination of f_y and f'_c.

Comparison with current design criteria

To illustrate the effects on development and splice lengths of both the newly proposed expressions and high relative rib area bars, values of l_d obtained with Eq. (22) through (25) are compared with development and splice lengths calculated under the provisions of ACI 318-95. Comparisons are limited to uncoated bottom-cast bars.

Eq. (22) through (25) differ from current design criteria in several important respects.

1. The relationship between l_d and the steel stress f_s or f_y is linear but nonproportional, rather than proportional, as in current design expressions. The more accurate representation provided by Eq. (22) through (25) results in values of l_d that are relatively shorter for f_y < 60 ksi (414 MPa) and relatively longer for f_y > 60 ksi (414 MPa) than obtained with ACI 318-95. Eq. (22) through (25) automatically account for the fact that, when f_y is increased by 25 percent from 60 to 75 ksi (414 to 517 MPa), l_d must be increased by more than 25 percent.

2. The effect of concrete strength on bond strength is represented by $f'^{\,1/4}_c$ rather than $f'^{\,1/2}_c$. The impact of this change is greatest for high-strength concrete. The proposed expressions apply up to at least 16,000 psi (110 MPa); the development length expressions in ACI 318-95 limit $f'^{\,1/2}_c$ to 100 psi (0.69 MPa), corresponding to f'_c = 10,000 psi (69 MPa).

3. Using Eq. (22) through (25), splice length and development length are identical, removing the requirement to multiply l_d by 1.3 (ACI) or 1.7 (AASHTO) to obtain the length of most splices.

The key aspects of the development/splice length criteria of ACI 318-95 are summarized next.

ACI 318-95—Under the provisions of ACI 318-95, two options are available for selecting development length. One involves a chart with selected expressions for l_d/d_b, and the other involves the use of a more detailed expression for l_d/d_b. Under Section 12.2.2 for bottom-cast uncoated reinforcement, $l_d/d_b = f_y/(25\, f'^{\,1/2}_c)$ for No. 6 and smaller bars and $f_y/(20\, f'^{\,1/2}_c)$ for No. 7 and larger bars if the bars have a clear spacing between bars $\geq d_b$, cover $\geq d_b$, and transverse reinforcement is not less than the code minimums, or clear spacing between bars $\geq 2\, d_b$ and cover $\geq d_b$. For all other cases, $l_d/d_b = 3\, f_y/(50\, f'^{\,1/2}_c)$ for No. 6 and smaller bars and $3\, f_y/(40\, f'^{\,1/2}_c)$ for No. 7 and larger bars.

Under Section 12.2.3

$$\frac{l_d}{d_b} = \frac{3}{40}\,\frac{f_y}{f'^{\,1/2}_c\left(\dfrac{c + K_{tr}}{d_b}\right)} \qquad (26)$$

in which $K_{tr} = A_{tr} f_{yt}/(1500\, sn)$, $(c + K_{tr})/d_b \leq 2.5$. Although K_{tr} is the same symbol as used in this study to represent the effect of transverse reinforcement, the value includes f_{yt} and does not correspond to the value in Eq. (21) through (23).

When 50 percent or less of the reinforcement is spliced at one location and the area of steel provided is equal to or greater than twice the area required, the splice length is equal to 1.3 l_d.

Bars not confined by transverse reinforcement—For bars not confined by transverse reinforcement, it is appropriate to compare the simplified expressions in ACI 318-95 with the development and splice lengths obtained using Eq. (24) and (25). For No. 7 (22-mm) bars and larger with clear spacing $\geq$ 2 d_b and cover $\geq d_b$ and 4000 psi (28 MPa) concrete, l_d/d_b is 47.4 for developed bars and 61.7 for Class B splices, under the provisions of ACI 318-95, and 52.26 using Eq. (24) for both developed and spliced bars. Thus, using the proposed expression, the development length is 10 percent greater than under the provisions of ACI 318-95, while the splice length is 18 percent lower. The same percentages hold for the conditions under which Eq. (25) is applied. Overall, for normal-strength concretes, Eq. (24) and (25) result in greater development lengths and shorter splice lengths than do the provisions of Section 12.2.2 of ACI 318-95. The increases in development length are more than matched by the reductions in splice length.

Comparisons of development and splice lengths obtained using Eq. (22) and (23) with the more detailed provisions of ACI 318-95 [Eq. (26)] are summarized in Table 2 for the 35 beam configurations used by Darwin et al. (1995c, 1996b) to develop the reliability-based ϕ factor [the detailed comparisons are presented by Darwin et al. (1995b) and in Appendix B].* The tables cover concrete compressive strengths of 3000, 4000, and 6000 psi (21, 28, and 41 MPa) for developed or spliced No. 6, No. 8, No. 10, and No. 11 (19, 25, 32, and 36-mm) bars. Comparisons show that development lengths obtained with Eq. (23) (the more simplified of the two new expressions) are, on average, 114 percent of those obtained with ACI 318-95. Development lengths obtained with Eq. (22) are, on average, 102 percent of those obtained with the Code. The splice lengths obtained with Eq. (23) average 88 percent of those obtained with ACI 318-95, while those obtained with Eq. (22) average 78 percent of those obtained with the Code. These comparisons show that Eq. (22) and (23) result in a small increase in development length and a substantial reduction in splice length compared to values obtained under the provisions of ACI 318-95.

Bars confined by transverse reinforcement—Comparisons of development and splice lengths obtained using Eq. (22) and (23) with those obtained under the provisions of ACI 318-95 are summarized in Table 2 for the 140 beams with transverse reinforcement used to develop $\phi = 0.9$ (Darwin et al. 1995c, 1996b) [the detailed comparisons are presented by Darwin et al. (1995b) and in Appendix B].* Comparisons include development lengths obtained with both conventional and high relative rib area reinforcement. Results in Table 2 show the following.

Effect of relative rib area. Limiting consideration to the effect of using high relative rib area bars (a savings not available under ACI 318-95), the average ratios of l_d for high relative rib area bars to l_d for conventional bars are 0.87 and 0.84 using Eq. (22) and (23), respectively. Therefore, depending on the expression used for the design, average reductions of 13 to 16 percent in development and splice length can be expected with the use of high relative rib area bars.

Comparisons with ACI 318-95. For conventional reinforcement, the development lengths average 95 and 116 percent for Eq. (22) and (23), respectively, of those obtained using ACI 318-95; the splice lengths average 73 and 89 percent, respectively. For high relative rib area bars, the development lengths obtained with Eq. (22) and (23) average 83 and 97 percent, respectively, of the development lengths obtained with ACI 318-95; the splice lengths average 64 and 75 percent, respectively, of the splice lengths obtained with ACI 318-95. Overall, significant savings can be obtained with a conversion to the new expressions. Even higher savings are available when Eq. (22) and (23) are used in conjunction with high relative rib area bars.

SUMMARY AND CONCLUSIONS

Test results for 133 splice and development specimens in which the bars are not confined by transverse reinforcement and 166 specimens in which the bars are confined by transverse reinforcement are used to develop an expression for the bond force at failure as a function of concrete strength, cover, bar spacing, development/splice length, transverse reinforcement, and the geometric properties of the developed/spliced bars. The expression is valid for concrete strengths between 2500 and 16,000 psi (17 and 110 MPa). Results are used to formulate design criteria that incorporate a reliability-based strength reduction (ϕ) factor that allows the calculation of a single value for both development and splice length for given material properties and member geometry.

The following conclusions are based on the analyses and comparisons made in this paper.

1. The relationship between bond force and development or splice length l_d is linear but not proportional. Thus, to increase the bond force (or bar stress) by a given percentage requires more than the percentage increase in l_d.

2. $f'^{1/2}_c$ does not provide an accurate representation of the effect of concrete strength on bond strength over the full range of concrete strengths in use today. Development/splice strengths are underestimated for low-strength concretes and overestimated for high-strength concretes.

3. $f'^{1/4}_c$ provides an accurate representation of the effect of concrete strength on bond strength for concretes with compressive strengths between 2500 and 16,000 psi (17 and 110 MPa).

4. The most accurate representation of the effect of transverse reinforcement on bond strength obtained in the current analysis includes parameters that account for the number of transverse reinforcing bars that cross the developed/spliced bar, the area of the transverse reinforcement, the number of bars developed or spliced at one location, the relative rib area of the developed/spliced bar, and the size of the developed/spliced bar.

5. The yield strength of transverse reinforcement plays no significant role in the effectiveness of the transverse reinforcement in improving development/splice strength.

6. Depending on the design expression selected:

a. For bars that are not confined by transverse reinforcement, development lengths average 2 to 14 percent higher than those obtained using ACI 318-95, and splice lengths

Table 2—Ratios of development and splice lengths obtained using proposed expressions to development and splice lengths obtained using ACI 318-95

		Development lengths		Splice lengths	
		Eq. (22) / ACI 95	Eq. (23) / ACI 95	Eq. (22) / ACI 95	Eq. (23) / ACI 95
35 beams without transverse reinforcement	Minimum	0.785	1.036	0.604	0.797
	Maximum	1.176	1.377	0.904	1.059
	Average	1.017	1.141	0.782	0.878
140 beams with transverse reinforcement, conv. bars*	Minimum	0.776	0.832	0.597	0.640
	Maximum	1.270	1.730	0.977	1.331
	Average	0.951	1.156	0.732	0.889
140 beams with transverse reinforcement, high R_r bars†	Minimum	0.622	0.719	0.479	0.553
	Maximum	1.127	1.405	0.867	1.081
	Average	0.826	0.973	0.635	0.749

		Development and splice lengths	
		High R_r† / Conv.* [Eq. (22)]	High R_r† / Conv.* [Eq. (23)]
140 beams with transverse reinforcement	Minimum	0.779	0.753
	Maximum	1.000	1.000
	Average	0.867	0.842

*Average R_r = 0.0727.
†Average R_r = 0.1275.

*The Appendix is available in xerographic or similar form from ACI headquarters, where it will be kept permanently on file, at a charge equal to the cost of reproduction plus handling at time of request.

average 12 to 22 percent lower than those obtained with ACI 318-95 for Class B splices (i.e., for a 1.3 modification factor).

b. For conventional bars confined by transverse reinforcement, development lengths average 5 percent lower to 16 percent higher than those obtained using ACI 318-95, while splice lengths average 11 to 27 percent lower than those obtained with ACI 318-95 for Class B splices.

c. For high relative rib area bars confined by transverse reinforcement, development lengths average 3 to 17 percent lower than those obtained using ACI 318-95, while splice lengths average 25 to 36 percent lower than those obtained with ACI 318-95. When confined by transverse reinforcement, high relative rib area bars require development and splice lengths that are 13 to 16 percent lower than required by conventional bars.

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NOTATION

A_b = bar area, in.^{2}

A_{tr} = area of each stirrup or tie crossing potential plane of splitting adjacent to reinforcement being developed or spliced, in.^{2}

c = $c_m + 0.5\, d_b$

c_b = bottom cover of reinforcing bars, in.

c_M = maximum value of c_s or c_b ($c_M/c_m \leq 3.5$), in.

c_m = minimum value of c_s or c_b ($c_M/c_m \leq 3.5$), in.

c_s = min (csi + 0.25 in., cso) or min (csi, cso), in.

c_{si} = one-half of clear spacing between bars, in.

c_{so} = side cover of reinforcing bars, in.

d_b = nominal bar diameter, in.

f'_c = concrete compressive strength, psi; $f'^{1/2}_c$ and $f'^{1/4}_c$, psi

f'^{p}_c = concrete compressive strength to power p, psi

f_s = steel stress at failure, psi

f_y = yield strength of bars being spliced or developed, psi

f_{yt} = yield strength of transverse reinforcement, in psi

K_{tr} = term representing effect of transverse reinforcement on bond strength. Value depends on stage of analysis and design expression in which it is used. $K_{tr} = 29.6\, t_r t_d A_{tr}/sn$ based on initial analysis. $K_{tr} = 35.3\, t_r t_d/A_{tr}/sn$ based on final analysis [K_{tr} (conv.) = 34.5 (0.72 d_b + 0.28) A_{tr}/sn for conventional reinforcement (average R_r = 0.0727); K_{tr} (new) = 53 (0.72 d_b + 0.28) A_{tr}/sn for new reinforcement (average R_r = 0.1275)]

= $A_{tr} f_{yt}/(1500\, sn)$ in ACI 318-95

l_d = development or splice length, in.

N = number of transverse reinforcing bars (stirrups or ties) crossing

n = number of bars being developed or spliced along plane of splitting

R_r = ratio of projected rib area normal to bar axis to product of nominal bar perimeter and center-to-center rib spacing

s = spacing of transverse reinforcement, in.

T_b = total force in bar at splice failure, lb

T_c = concrete contribution to total force in bar at splice failure, lb

T_s = confining steel contribution to total force in bar at splice failure, in lb

t_d = 0.72 d_b + 0.28, term representing effect of bar size on T_s

t_r = 9.6 R_r + 0.28, term representing effect of relative rib area on T_s

β = reliability index

ϕ = reliability-based strength reduction factor

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